DOCUMENT RESUME

ED 312 127 SE 050 894

TITLE	Mathematics: Grades 1 and 2 Assessment.
INSTITUTION	North Carolina State Dept. of Public Instruction, Raleigh.
PUB DATE	Feb 89
NOTE	165p.; Drawings may not reproduce well.
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01/PC07 Plus Postage.
DESCRIPTORS	*Educational Assessment; *Elementary School Mathematics; *Evaluation Criteria; *Evaluation Methods; Grade 1; Grade 2; Mathematical Concepts; *Mathematics Materials; Mathematics Skills; *Mathematics Tests; Primary Education
IDENTIFIERS	National Council of Teachers of Mathematics; *North Carolina

ABSTRACT

This document is the result of an extensive effort to design an assessment program in communication skills and mathematics for first and second graders. It contains resource materials and instruments that school systems can use in their assessment programs. The resource materials include strategies for teachers to use in evaluating student performance on different curriculum goals. The first section provides an overview of mathematics assessment including beliefs, implementation and curriculum integration. The next two sections describe assessment strategies for seven competency goals for grades one and two respectively. The appendices include: (1) "Grade 1: Competency Goals and Objectives"; (2) "Grade 2: Competency Goals and Objectives"; (3) "Sample Profile, Grade 1"; (4) "Sample Profile, Grade 2"; (5) "Assumptions from NCTM 'Standards'"; (6) "K-4 NCTM 'Standards'"; (7) "NCSM Essential Mathematics"; and (8) "Mathematics Division Staff." (YP)

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MATHEMATICS

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SE050 894

Grades 1 and 2 Assessment

Bob Etheridge, State Superintendent

North Carolina Department of Public Instruction

Febuary, 1989

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Foreword

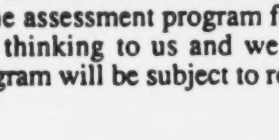
In recent years the General Assembly has supported our efforts to improve the quality of education in North Carolina by enacting legislation and providing funding to accomplish our goals.

The Elementary and Secondary School Reform Act of 1984 directed the State Board of Education to undertake an audit and revision of existing curricula in order to develop a standard course of study. Funds appropriated to implement the reform act were used to produce the North Carolina Standard Course of Study and the accompanying Teacher Handbook for all subjects at all grade levels. These documents identify the curriculum that must be available to all students in the state's public schools.

In 1985 the General Assembly stressed in Senate Bill 1 its intent that every child master a common core of knowledge and skills defined in a basic education program before graduation from high school. A plan was established to appropriate expansion monies for phased-in funding until 1993, when the program will be fully implemented. The publication that ensued, the Basic Education Program, not only contains curriculum descriptions that summarize the Standard Course of Study but identifies minimum competencies that must be mastered before a student can be promoted from grades three, six, and eight. The General Assembly has continued its commitment to funding the Basic Education Program.

Legislation that mandated state-wide testing of basic subjects in grades one, two, three, six, and eight was rewritten in 1988. The present legislation deletes grades one and two from the state testing program. Additional wording directs: "The State Board of Education shall also adopt and provide to local school administrative units developmentally appropriate individualized assessment instruments consistent with the Basic Education Program for the first and second grades, rather than standardized tests..." In response to this legislation, staff members in the Department of Public Instruction designed an assessment program for grades one and two in the basic areas of communication skills and mathematics. The program was piloted during the fall of 1988 in eight schools across the state and revised according to recommendations from the pilot sites. It was then approved by the State Board of Education in February, 1989.

Although the program has gone through extensive review and alteration, we expect you to have further suggestions to improve the quality of the assessment program for students in grades one and two. We encourage you to submit your thinking to us and we will give it careful consideration. It is our view that the assessment program will be subject to refinement as teachers use it and determine changes that will strengthen it.



Bob Etheridge
State Superintendent
Department of Public Instruction

Preface

The materials in this notebook are the result of an extensive effort to design an assessment program in communication skills and mathematics that is consistent with the Basic Education Program and reflects an instructional program that is developmentally appropriate for first and second graders. Staff in the Department of Public Instruction used current research to produce a draft of the program that was piloted during the fall of 1988 in eight schools across the state:

. Bath Elementary School, Beaufort County
. Southport Elementary School, Brunswick County
. Aycock Elementary School, Vance County
. Alma Easom School, Cumberland County
. Guy B. Teachey School, Asheboro City
. Central School, Albemarle City
. W. M. Jenkins School, Hickory City
. West Marion School, McDowell County

The materials were then revised, based on recommendations from the pilot sites. We are grateful to our colleagues in both the Department of Public Instruction and the pilot sites for their efforts.

In this assessment program, teachers are involved in on-going assessment of student progress. They periodically record information and comments about each child's level of functioning within the curriculum so instructional decisions can be made for that child. Teachers' recordings will be guided by their observations of children in normal classroom activities and by their evaluation of samples of each child's work. It is not intended that the recordings be made on all children in the class on the same day nor that recordings on all the items on a child's profile be made the same day. The program provides a structure and framework within which teachers use their professional judgements.

The notebook contains resource materials and instruments that school systems may use in their assessment program. The resource materials include strategies for teachers to use in evaluating student performance on different curriculum goals. In the area of communication skills, three options, with accompanying instruments are provided in order to meet the need for local school system flexibility. In the area of mathematics, the assessment involves the use of two instruments: one for first grade and one for second grade. All instruments that have been developed are consistent with the North Carolina Standard Course of Study and the Teacher Handbook.

Staff members in the Department of Public Instruction have designed staff development to assist school systems in the implementation of each part of this assessment program for grades one and two. Pleas : feel free to call on them as needed.

Joseph B. Webb
Office of Program Services

Acknowledgements

With the implementation of this Mathematics Assessment in grades one and two, North Carolina is opening the door to improved mathematics instruction. As evaluation and instruction are more closely aligned, teachers are able to put into place a broader, richer curriculum for all primary children.

The Mathematics Division extends thanks to the many North Carolina teachers, supervisors, and assistant superintendents who carefully studied, piloted, edited, and proofed the grades one and two assessment materials. This is truly a state-wide project. Teachers in the 1988 Summer Leadership Institutes at Meredith College and the University of North Carolina at Asheville provided the initial structure for the documents; K-3 teachers joined Jeane Joyner and other members of the Mathematics Division staff in writing the assessment strategies; and teachers in the eight pilot schools provided the input for the revisions.

We thank also the many friends in the mathematics education community across the United States who thoughtfully read and commented on the pilot materials. Children in North Carolina are the benefactors of this assistance.

Nothing in education must be allowed to remain static. As more teachers work with these assessment materials, improvements will be made. We invite and encourage your suggestions. Through staff development, parent involvement, and community awareness the Department of Public Instruction is striving to support teachers in the mathematics education of all North Carolina's children.

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MATHEMATICS ASSESSMENT: AN OVERVIEW

Throughout North Carolina there is a strong emphasis on quality mathematics education. Statewide expectations for the instructional programs, which are detailed in the Standard Course of Study and in the Teacher Handbook for Mathematics, form the basis of a mathematics curriculum which focuses on the development of understanding as well as on the acquisition and application of skills. With the removal of norm-referenced testing at grades one and two and the continued financial support for individual evaluations, the North Carolina Legislature has created an opportunity for primary educators to implement assessment instruments which more closely match the pedagogy and instructional methodology promoted by the Standard Course of Study.

Developing Mathematical Concepts

During the primary years students need to become confident in their abilities to do mathematics. They need time to understand fundamental ideas of mathematics before they are asked to apply these ideas symbolically. They need to perceive themselves as good problem solvers and to learn to reason mathematically. Just as language is a key to reading, developing mathematical language is crucial to success in mathematics. *Because concepts are developed over a broad period of time through mutiple experiences, mastery of an idea must not be assumed because of correct responses given in one setting.* Primary students need time to assimilate ideas as they explore mathematics.

Rushing students to complete the textbook or to prepare them for a standardized test has resulted in students entering school the subsequent fall having "forgotten" what they appeared to have mastered the previous spring. Teachers spend a great deal of time each year reviewing material and reteaching rules and procedures because learning has been superficial. *When students have opportunities to apply their ideas in many contexts and to construct their own understandings rather than memorize rules, they begin to internalize mathematical concepts. These are the concepts upon which they can rely when they must work with abstract notation.*

Developmental stages do not begin and end at fixed times. Within every classroom there will be children needing more exploratory opportunities for a concept, students working on internalizing the concept, and other children who are ready to extend their understanding of the concepts being examined.

Teachers must continually help students to make the connection between their explorations with concrete materials and the symbols used to record mathematical ideas. Concepts should be explored in many contexts with experiences from all strands interwoven throughout the years. As students study topics in measurement and geometry, for example, they must relate the new ideas with those they explored in numeration and patterning lessons. Mathematics should also be integrated with other content areas as concepts are applied to real life situtations. Inconsistencies in individual student's performance diminish with age and with a wide variety of experiences. As teachers help children see the connections between mathematical ideas and focus on the relationships of the manipulative materials and symbolic notations, students demonstrate the consistency which comes with mastery.

Mathematical Thinking

An understanding of mathematics allows students to communicate ideas both verbally and symbolically, to develop insight and to reason, to solve problems, and to have confidence in themselves as learners. The use of manipulatives, opportunities for cooperative problem solving, and lessons in which students are actively involved with mathematics are essential to the development of mathematical thinking and reasoning. Success in mathematics in later years depends to a large degree upon the quality of the foundations laid during the primary years.

To encourage the use of manipulative materials for developing concepts North Carolina has three levels of manipulatives kits. However, the lack of a kit or a special material is not a reason for introducing concepts at abstract levels. The specific manipulatives are not the important aspect of the lesson but rather what students internalize from their experiences. Different students will benefit from "hands-on" lessons in different ways depending upon their previous experiences, interests, and levels of understanding. *Without experiences which lead to understanding of mathematical ideas, the symbols used to describe mathematics at the abstract level are meaningless. If students do not understand the concepts which symbols are representing, practice with the symbols will not help to develop the concept.* For example, when a child solves the number sentence 5 - 2 and incorrectly writes 5 as the solution, it may be the result of a logical strategy. The child is "taking away" the 2 by covering it. When you cover the 2, the 5 is left!

Just as previously learned concepts and skills must be continuously reinforced, consistent attention must also be given to developing children's problem solving abilities. Students need opportunities to define problems, to identify needed information, to explore strategies, to test their ideas, to search for alternate solutions, and to work with other children, listening to others and explaining their own thoughts. They must develop confidence in themselves as critical thinkers and problem solvers in the very broadest definitions of the terms. Technology is rapidly changing our lives; students must be able to decide what mathematics they need, how to apply it, and how to evaluate possible solutions. Memorization of algorithms alone is not sufficient. Problem solving strategies, estimations, and mental computations must be included in the curriculum at every level. *Children's mathematics must not be limited to what they are able to record with symbols .*

Beliefs

Beliefs about mathematics, about teachers' judgment as professionals, and about the ways in which most children learn provide the foundation for these first and second grade assessments. These beliefs may be summarized in five statements:

1. **Concepts develop over a period of time through a variety of experiences. Children should be exposed to ideas at least a year before they are expected to deal with them independently.**

2. **Language - naming, explaining, restating, and describing - is the bridge from concrete to symbolic. Students need opportunities to tell what and how they think about mathematics.**

3. **Young children have greater thinking and reasoning capacities than skills with pencil and paper tasks demonstrate. Children are denied a rich, stimulating mathematical environment if lessons are dependent upon symbols only.**

4. **Mathematics lessons should communicate high expectations for student involvement and understanding. Teachers must structure classes to insure feelings of success and to encourage students to question.**

5. **Children's mathematical thinking evolves in different ways and at different rates. Assessments should focus on each child's growth of knowledge and skill. Teachers' professional observations of student performance made over a period of time provide a more reliable evaluation of student achievement than a single test.**

The Teacher Handbook has established goals and objectives to provide guidance in instructional planning for students, not to define success and failure for an individual child. Teachers who observe students' performance on a day-to-day basis are the ones best prepared to give a professional evaluation of students' mathematical knowledge. Through on-going observations teachers are able to describe students' understandings and their abilities to demonstrate more complex applications of previously mastered skills. Progress is viewed as continuous, and there is the expectation that all children will grow in knowledge and mathematical understandings.

Mathematical ideas build one upon another, but not in a totally linear fashion. For example, sudents are developing spatial visualization skills at the same time they are learning about numberness. Measurement activities lead to an understanding of comparative relationships at the same time that children are exploring patterns. Nurturing students' abilities to communicate mathematical ideas, not necessarily in abstract symbols, and to apply mathematics to new situations is critical and must not be subsumed by an over emphasis on rote drill. Students' understanding of mathematical concepts must be more important than their memorization of procedures or definitions that lack meaning beyond the immediate lessons.

Purposes of the Assessments

At every level assessments of students' understanding of concepts and skills should provide the teacher with information for planning further instruction. This is the primary purpose of these evaluation instruments. *Whether an informal or a formal evaluation, assessments should focus on students' growth (progress) in acquiring new skills, abilities to reason, and greater understanding.* In recording observations, teachers must acknowledge movement toward more complex thinking levels and increased abilities to apply mathematical thinking to new situations.

A second purpose of these instruments is to provide teachers and parents with a profile of an individual student's performance as related to specific mathematical concepts and skills at first and second grades. Students' strengths and needs are noted without a specific score being attached to

the performance. Children are not compared on a pencil and paper test to a national standard which tends to focus on computational prowess but rather are encouraged to explain how they obtain solutions. On some objectives the teacher may note the level (or range of numbers) where the student is working. Teachers will also be able to indicate when children exhibit an understanding of more complex applications of skills or ideas.

The third purpose of these assessments is to match methods of instruction with methods of evaluating student progress. By more closely aligning assessment with instructional practices, teachers should have time to focus on the conceptual development of mathematical ideas rather than dwell on rote drill with abstract symbols. *While being able to record mathematics through symbols is certainly an appropriate goal which should not be excluded from K-2 classrooms, mathematical understanding through active learning should be the primary concern.*

A final purpose of these instruments is to lighten the demands on teachers' professional time by formalizing information which teachers are already informally compiling. Strong primary instruction has always involved on-going assessment and focused on individual students' growth. By providing a structure and format with a record keeping system consistent with competencies in the Teacher Handbook and the Standard Course of Study, teachers will know what the mathematics program should be, will be able to create a profile of each child's mathematics, and will be able to plan instruction accordingly.

Format and Implementation

At grades one and two there is a folder for each student which details mathematical expectations. For each objective on the folders there are two spaces for teachers to make comments or to indicate student achievement in a checklist format. The folders also serve as files for saving samples of students' work. Every teacher using the assessment profiles will have a copy of this manual which gives examples of ways to assess each of these competencies. The manual includes management suggestions, sample worksheets, individual and group activities, and record-keeping ideas which teachers may (or may not) choose to use. Discussions of the objectives with examples of ways to assess each item will clarify the brief statements found on the profiles.

The assessment profiles are designed to be completed toward the end of each semester after teachers have made observations over a period of time. *It is not an expectation that there will be a formal time set aside for testing nor that there will be written comments for every objective for every child.* A checklist scoring may be used whenever appropriate. The suggested notation

printed on each profile is "N" for "Not yet", "S" for "Sometimes", and "M" for "Most of the time" (See Figure 1). Teachers who wish to use another form of scoring, for example " - ", " √ ", and " + ", should indicate their preferred notation on the profiles. The date that the notation is written and the single symbol code may be placed in the small rectangle, leaving the larger space for annecdotal comments.

Assessing the mathematical progress of each child does not mean that each child is tested individually by the teacher or the assistant. Students should be observed as an integral part of the daily instructional program. The observations and recording should take place at the teacher's convenience. Because of the varying developmental levels of students, some items may not be appropriate for all children when they are introduced to the class. The teacher's anecdotal comments would explain these situations.

7.3 Model subtraction as take-away; As comparison	*11/8* *M*	*Still has some difficulty with comparison*		
7.4 Create and solve problems using addition/subtraction	*1/7* *S*			

Figure 1

The structure within the assessment process comes through the teacher's planning. For example, a teacher may decide to focus on the students' patterning skills, their abilities to create, translate, and describe patterns, and their abilities to recognize patterns in the environment and in other content areas. During the same lessons there will be additional opportunities to note strengths and weaknesses in individual's use of directional and positional words, ordinals, and visual memory. While the lessons may be conducted in large or small groups, the teacher will be focusing on a few children at a time.

The order of the competencies does not imply that children must master all numeration indicators before geometry or problem solving or other concepts are introduced. While place value activities logically come after children understand the numbers 0 to 9, patterns and shapes and directional ideas should be interwoven with other strands. *Integrated curriculum means not only applying mathematics in other content areas but also focusing on the interrelatedness of different strands in mathematics.*

The competency goals and objectives listed on the student profiles form the basic core of the mathematics program for all first and second graders in North Carolina. While many school

systems may add requirements beyond those which are outlined, no child should be denied the opportunity to experience these broad goals. It is the expectation that many students will move beyond the basic mastery of these objectives into more complex applications while other children will exhibit inconsistent performance as they move toward mastery. By assessing student progress through continuous observations and samples of children's work, teachers and school administrators will have a clearer picture of what an individual child understands, the amount of progress the student has made, and areas of continued concern. This detailed information will be helpful in parent conferences and in preparing report cards.

Toward the end of each semester teachers will record their observations on the student profiles for objectives that have been taught. *Some objectives from each of the seven goals will be taught during the first semester* (with the strongest focus being goals 3, 1, 5, 6, and beginning 7). In the second semester many objectives will continue to be taught in more complex lessons as well as new ones introduced. Teachers should make comments about the levels of performance, noting how students approach different tasks. For example, given objects to classify, some students will group only by very obvious attributes such as color or shape. Other students might demonstrate a more complex level of thought by grouping the objects by their uses or by subsets of the obvious groupings.

It is the intent that teachers make the majority of their observations as students are working in groups or alone on day-to-day lessons. No special times need be set aside for "testing" unless the teacher wishes to examine an individual's performance more closely. Pencil and paper tests found in all state adopted mathematics textbooks may be used to supplement the teacher's annecdotal records.

Assessment Strategies

The latter part of this teachers' manual contains a list of the competency goals and objectives with suggestions for ways of assessing students' performance. There are several suggestions for each objective so that teachers may observe students at more than one task. *Because young children waver between developmental stages before fully achieving a level, teachers will want to observe students in several situations rather than assuming that the response to a single item is representative of the child's understanding.* Just as instruction should focus on activity oriented lessons, the majority of the competencies should first be evaluated with concrete objects. The examples of ways to assess each competency reflect concrete, pictorial, and abstract levels of the mathematical ideas whenever appropriate. Teachers' professional judgment about the cognitive level at which a child is functioning and the manner in which a child works are important indicators

for evaluation. Comments should be made whenever they are needed to clarify a child's record.

The majority of the assessment strategies are written in terms of an individual child. This is not to imply that children must be evaluated in an individual setting; rather, the wording is designed to help focus on the behavior being evaluated. The majority of the tasks could be group or total class activities with the teacher focusing on specific children. For some objectives a few activities are included that do not lend themselves to evaluation but are good experiences for children .

It is not the intent of these documents to cause teachers to duplicate records. The basic mathematics competencies for each grade level are given in the Teacher Handbook; these competencies should guide the planning for every classroom's mathematics instruction. If local education agencies have additional criteria, they may be added. *Because this assessment combines developmental expectations for young children with high expectations for acquiring mathematical concepts and skills and problem solving abilities, instruction and testing will go hand in hand.*

Modifying Instructional Practices

An exciting part of the process of implementing a new assessment program for first and second grades across North Carolina is the opportunity to restructure the manner in which mathematics is being taught. Lessons must involve "hands-on" experiences, active participation in experiments and graphing activities, discussions of group solutions, and opportunities for children to do and talk mathematics. The focus on language is strong throughout both grade levels. As students work, the teacher must question their activities, their thinking, their alternate solutions, and the relevance of previous experiences to the current lesson. Teachers will spend less time telling and more time asking; students will spend more time doing and explaining and less time working in isolation on worksheets. While some lessons remain drill and some remain very structured, others will provide opportunities for students to work on a variety of problems. Classrooms will require manipulatives, necessitating a commitment on the part of school administrations to provide appropriate materials. Not all students must do all problems or even the same problems. Many lessons will take more than one day to complete, and all lessons would have the potential of incorporating a variety of mathematical ideas and concepts. Children will experience the real-life skills that are a part of working cooperatively with a partner or in a group.

It must be clear that deemphasizing rote memorization and worksheets which provide drill at abstract levels with no reference to application does not imply a less rigorous curriculum. The opportunities for initiating positive changes through this matching of teaching practices and evaluation stategies mean that children will experience a broader, richer curriculum - not one that

focuses on only what can be easily tested with pencil and paper. Instructional objectives will remain clear; activities must continue to be orderly and purposeful; and expectations for mastery will remain high. Teachers will be encouraged to sharpen observational skills, focusing on processes, analyzing misconceptions and errors, and recognizing the indicators of success.

Integrating the Curriculum

Discussions of integrated curriculum tend to focus on combining content from two or more disciplines. Common examples include mathematics applications in science and social studies, writing assignments in which the topics come from reading lessons or classroom activities, or social studies units which include literature selections and oral reports. Little has been written, however, in North Carolina that focuses on integrating curriculum within a content area.

Fragmenting mathematics into separate topics, taught in isolation from real-world applications and segregated from each other, tends to lead to a curriculum of memorization without meaning and achievement at levels lower than students' potentials. Specific objectives, which are needed to describe and illustrate the broad goals of "understanding", have consistently been misused. Rather than serving as descriptors and guidelines for teachers in planning instructional programs, the objectives have been taught in isolation as goals within themselves. Mathematics, when so narrowly defined and practiced by having students work silently and alone at their desks, does not become the powerful tool needed by all citizens. For example, computation lessons without a relevant context offer little to discuss, and problem solving activities which focus on one correct response and one method of finding the answer do not lead to explanations of thinking and reasoning.

In planning lessons teachers must capitalize on the interrelatedness of mathematical ideas. For example, the lesson which explores different triangles made on a geoboard begins as geometry but becomes an exercise in classification of triangles when the geoboards are grouped by the sides of the triangle or by the type of angles (corners). It extends to problem solving and logical thinking when the teacher asks children to explain why all triangles may not look the same or if it is possible to make a triangle with more than one right angle. Spatial visualization skills are employed when students record their triangles on dot paper.

Another example is the pattern block lesson which has the primary purpose of spatial problem solving and logical thinking but begins with a spatial visualization and estimation task. Given an outline (a picture or an irregular shape), students are asked to estimate the number of pattern blocks it will take to fill the puzzle. Would the estimate be the same if only one color of blocks is used?

After completing the puzzle, each student is asked to record the number of blocks used (first sorting, then counting, then recording). What is the total number of blocks used by each student? Was the total close to the estimate? Students are directed to graph the number of blocks each person used and the basic information on the graph is discussed. The class graph could be saved for a later addition or multiplication activity using calculators. The lesson continues with students being asked to complete the puzzle with a different number of blocks. Tell your partner if you plan to use more pieces or fewer pattern blocks. Students sort, count, and record their second solution.

Computation becomes a part of the next day's lesson as the teacher gives a value to each pattern block piece. For example, hexagons are worth 5¢; trapezoids are valued at 3¢ each; squares cost 4¢; blue parallelograms are worth 2¢; and triangles and tan parallelograms are the least expensive at 1¢ each. What is the most expensive way to cover the area of one hexagon? How many different ways could you cover that area and what would each cost? If a student had to purchase the pattern block pieces to make the pictures, which version would cost the most? Could you find a way to cover the puzzle which would be less expensive than your first two versions?

Money becomes a more important part of the lesson as students count out the coins to "purchase" blocks for their puzzles. What would be alternate sets of coins that would have the same value? Suppose there was a sale on blue parallelograms and, instead of costing 2¢ each, they were marked 5 for a nickel. Would the cost of the picture change? Would the design which uses the fewest blocks also be the least expensive design? Working with partners determine the most expensive way to complete the picture.

Oral and written language skills are used as students tell and write about their explorations with pattern blocks or their experiences with making triangles on the geoboard. Vocabulary and articulation skills increase as students explain their thinking and reasoning. Language is an integral part of the lesson and is not an add-on assignment. As teachers observe students at work, talk with them, listen to their conversations, and check their written work, assessment should be taking place. When students create or interpret graphs in science or social studies lessons, teachers should be assessing the application of mathematics competencies.

Trends in Mathematics Education

Contained in the Appendix are summaries of documents which reflect the thinking of many of the leading mathematics educators across the nation. The National Council of Teachers of Mathematics and the National Council of Supervisors of Mathematics have established what they

believe are guidelines for mathematics as education moves into the next century. Philosophically and professionally, the majority of mathematics educators in North Carolina have aligned themselves closely at the elementary level with the work and the attitudes reflected in these documents. This first and second grade assessment program is one example of how some of the ideas are translated into reality.

The first summaries are from the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics). These detail underlying assumptions about mathematics and its learning by children and then present thirteen curriculum standards for grades K-4. Also included is a summary of recommended changes in the content and emphasis of K-4 mathematics. Copies of the entire Standards document may be obtained from the National Council of Teachers of Mathematics after April 1989 for $25.00. The mailing address in 1906 Association Drive, Reston, Virginia 22091.

"Essential Mathematics for the 21st Century" is the position paper of the National Council of Supervisors of Mathematics. In the Appendix is a copy of the twelve components of a mathematics program which NCSM considers essential for all students for the next century.

The NCTM Standards and the NCSM "Essential Mathematics for the 21st Century" both lend support to the mathematics program envisioned in the Standard Course of Study and the Teacher Handbook. In classrooms which focus on understanding concepts, communicating effectively, solving problems efficiently, and applying mathematics in the world around them, students will enjoy a firm foundation upon which to build further mathematical knowledge.

MATHEMATICS

GRADE 1

ASSESSMENT STRATEGIES

Completing Student Profiles

In recording observations about an individual student's knowledge of mathematical concepts and skills, teachers should not set aside specific "testing" time for the majority of the objectives. The purpose of ongoing assessment is to observe the student using skills in a variety of settings. Only for screening special cases or for noting objectives that are difficult to observe informally would individual or small group sessions be needed.

Initial observations should be recorded during the first semester for skills which have been taught. The suggested code for the profile is "N" for not yet, "S" for sometimes, and "M" for most of the time. Figure 1 explains the form.

Goal 1: *Broad goals are printed for each section of the profile*

1.4 Objectives to clarify-- the competency goals are printed in left-hand boxes	*Dates and codes*	*Comments reflect observations during the first part of the year ... included as appropriate*	*Dates and codes*	*Comments reflect observations during the latter part of year... clarification of performance*
1.4 Identify ordinal position				

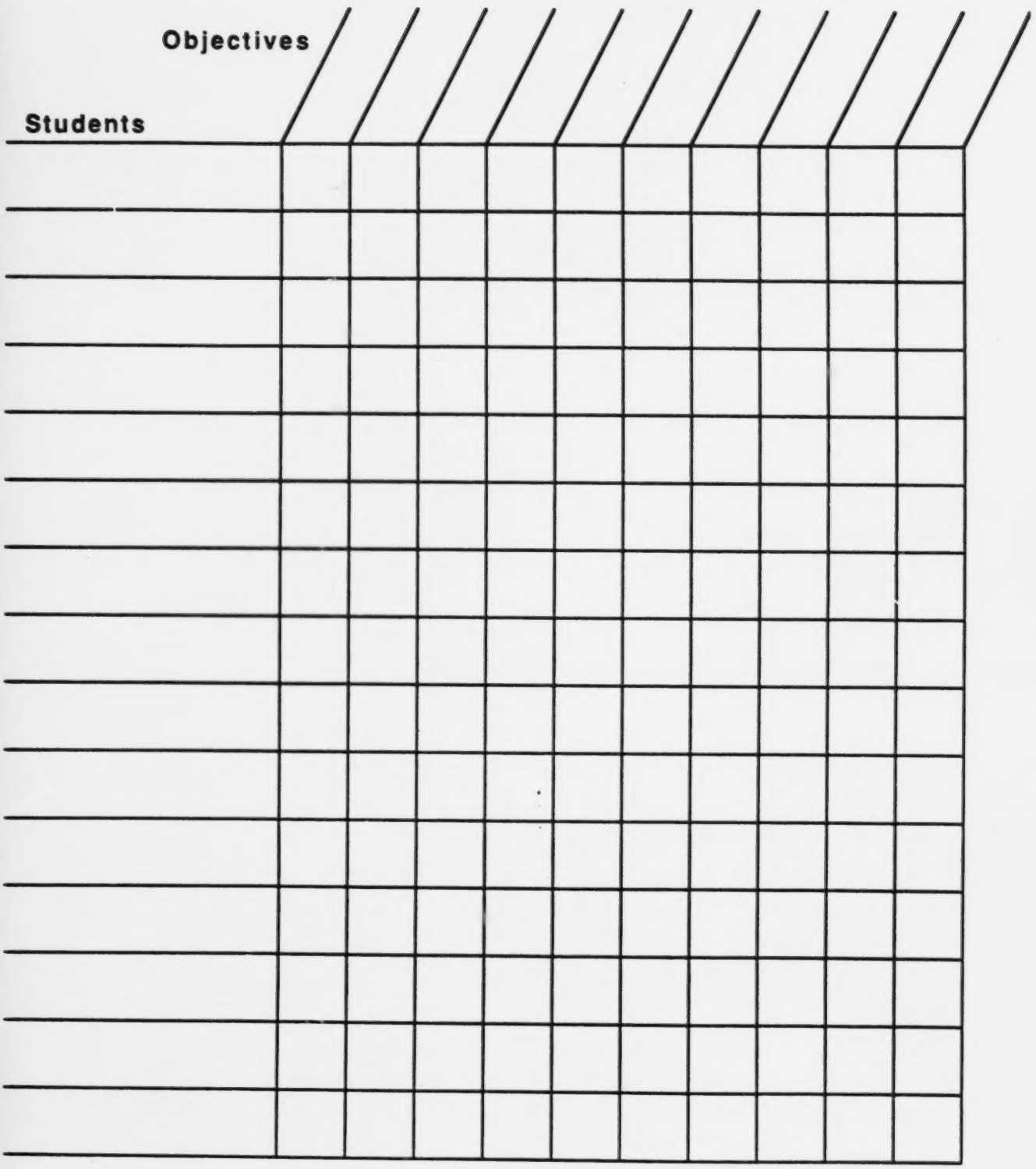
Figure 1

For each competency goal and objective, there are several suggested assessment strategies. While the suggestions are stated in terms of the teacher working with an individual to complete a task, the student most frequently will be working with other students. The teacher might ask a large group of students to complete a task and note specific students' performance. If lessons are presented to the total class and there are objectives that are not developmentally appropriate for some students at that point in the year, teachers should make appropriate notations. Similarly, if a student demonstrates more advanced understandings, an earlier and more detailed look at the child's performance will be helpful in instructional planning.

While primary teachers have always evaluated their students based on daily performance, recording these observations on a standardized checklist which relates directly to the Teacher Handbook will create a record of student understanding and progress that will be helpful in planning instruction as well as parent conferences. Samples of students' work will help illustrate notations on the profiles.

The final comments, recorded late in the school year (or whenever individuals exhibit consistent achievement of an objective), should attend to a student's independent efforts . Some objectives have appropriate oral or "hands-on" indicators, while others are appropriate pencil and paper tasks and will be assessed in written form. Chapter, unit, and summary tests from the state adopted mathematics textbooks may be used to supplement the teacher's observations. Many students will go beyond expectations in some goals and must never be denied opportunities for continued growth and challenge. While there will not be numerical scores for students, the profiles should show the extent of students' achievements in different aspects of mathematics.

Group Profile



GRADE ONE - ASSESSMENT STRATEGIES

Competency Goal 1: The learner will identify and use numbers, 0 to 100.

1.1 Count using 1 to 1 correspondence

Many children are inconsistent in counting objects accurately. Early, prenumber experiences use one to one correspondence for comparing without counting. Because accurate counting is necessary for many tasks , teachers should model strategies to help students master the task. A divided workmat or a container into which objects may be counted are examples of ways to help organize the counting.

A. Place 6 objects on the desk. Ask the child to count them. If a child has difficulty, try fewer objects. Increase the number of objects as appropriate. If a students counts up to 15 or 20 objects consistently, there is no need to go further.
B. Ask student to count out enough scissors for each child in his math group. How do you know you have enough?
C. Given some spoons and fewer forks, ask the child if there are as many forks as spoons. How many more do you need to have the same number?
D. Have child count number of objects in a picture.
E. Play musical chairs. Ask a child to explain how the game works (i.e. not enough chairs for each child to have a seat).
F. Have students "count off" with each child stating the appropriate number.

An understanding of number is essential for all children. Numbers are used to describe quantities and to show relationships between the quantities. Number is a concept; numerals and other symbols are labels for ideas and quantities. Rote counting helps children learn number names and sequences, but experiences with real objects are necessary to develop meaningful concepts of number that eventually allow children to think of groups as if they were individual items. Patterns which form the basis of our place value system allow us to deal with large numbers using the same 0 to 9 digits that first labeled how many in a set. If children are not yet conserving large numbers, the relationship, for example, of thirty-two loose objects, three tens and two extra cubes, and 32 may not be obvious. Children who memorize rules and parrot responses without understanding lack a foundation upon which to build other

complex concepts. While rote counting and other drills have important places in the classroom, primary teachers must give their students the gift of experiencing mathematics.

1.2 Make sets; Match with numerals

In learning to create sets and make appropriate connections with the oral names and symbols students need opportunities to experience the relationships in several important ways. The goal of this objective is for students to be successful with all variations of the task:

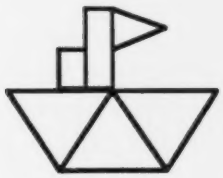
a. Shown a set, the student can tell how many .
b. Shown a set, the student can identify the appropriate numeral.
c. Shown a numeral, the student can state its name.
d. Shown a numeral, the student can create a set.
e. Given an oral name, the student can identify the numeral.
f. Given an oral name, the student can create a set.

A. Ask student to give you five (or another number) crayons; ask child to show you the numeral five. Show a student six crayons; ask child how many; ask child to match the correct numeral with the set..
B. Play "Huddle in groups of . . ." Leader calls out a number or shows the number on a large card. Students arrange themselves into appropriate groups. Repeat the activity with different numbers. Ask individual students to check if the groups have the correct number.
C. Tell student to use a specific number of pattern blocks to use in making a design. Direct child to find a number card to show how many blocks were used.
D. On a worksheet with pictures of objects, have student match or write the proper numerals. Fold to divide a sheet of newsprint into four parts and ask student to draw a given number of objects in each section.

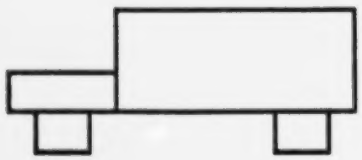
1.3 Compare and order sets

More than and less than are difficult ideas because, like other comparative words, they are relative. Six is more than four, but it is less than eight. Because "less" (fewer) is more difficult for most students than ideas of "more", children need additional opportunities to explore relationships that focus on less.

A. Child uses blocks to complete puzzles and tells which puzzle has more (or less, or fewer, or the same).



"This picture has more blocks"



"This one has fewer blocks."

B. Two or three students draw number cards and build towers of unifix cubes. They compare to see who has more or less (fewer) or if they have the same. Students order towers from least to greatest.

C. Teacher makes a set and directs student to make another set with more (less, fewer, or the same) than the teacher's. *For a more difficult task give the child a number and ask him to make a set that has more or less. This is difficult for many children and should be experienced many times.*

D. Given containers (workmats) with objects, student orders them from least to greatest. Explain how you decided the correct order?

E. Child is shown pictures of objects and asked to tell which groups have more (less, the same). Child can order pictures from least to greatest.

1.4 Identify ordinal position

A. Child takes part in acting out finger plays and poems which use ordinal numbers. For example, "Five Little Squirrels" or "Ten Little Jack-O-Lanterns". Who is first? Which student is fifth?

B. When shown a row of objects, child will name (or identify) which ones are in given ordinal positions. *When using ordinal numbers, always identify where the sequence begins (i.e. tell what is first in line).*

"Which block is second in line? "

"Which block is fifth?"

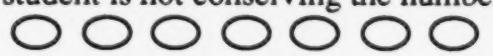
C. Child identifies ordinal position of objects (characters) in pictures.

D. Make a line of children and ask who is in different ordinal positions. Have some students sit down or change places in line. "Now who is in fifth place?" *Many children who are able to identify ordinal positions when shown sequences have difficulty applying the concept.* Have child get into line in order to be the third (or seventh, etc.) student.

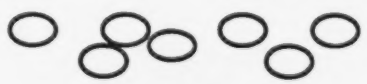
1.5 Conserve numbers

Conservation is the ability to mentally hold some property of a substance constant while other properties undergo change. For example, conservation of number is the understanding that the number of objects does not change if the same counters are rearranged in some manner. There are basically four steps in assessing a child's level of thinking through a conservation task. The student must demonstrate (explain) that two sets are equivalent. Secondly, one of the sets must be rearranged in full view of the student. The student must evaluate the new situation (i.e. determine if the sets are still equivalent). Finally, the child should be able to explain (justify) the answer. Because conservation of larger numbers continues to develop throughout primary years, internalization of "eightness" does not necessarily imply conservation of 2-digit numbers.

A. Teacher has student place a number of counters on work mat. Child counts to confirm how many. Teacher rearranges the counters as student watches and asks student how many there are? If child recounts, the student is not conserving the number.



"Put 7 counters on your mat."



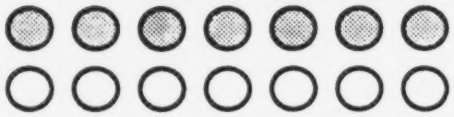
"How many are there now?"

B. Child places a number of objects into one of the teacher's hands. The teacher asks how many. The teacher then divides the same objects, placing some in each hand and asks, "How many do I have now?" Recounting indicates a lack of conservation.

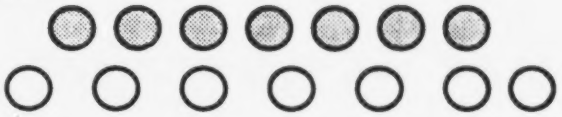
"Put six counters in my hand."

"How many do I have now?"

C. Child is asked to make two rows of counters with the same number in each row. Teacher asks child to verify that there are the same number in each row. While student watches, teacher then extends one row by spacing the counters farther apart and asks student if there are still the same number in each row. If the child thinks that the longer row has more, the student does not conserve.



"Are there the same in each row?"



"Are there the same in each row now?"

D. Teacher places 9 cubes on the table. Have student count the cubes. Ask student to snap a different color cube on top of each one of the teacher's. Then teacher unsnaps cubes and arranges them in sets by color. The physical arrangement of each set should be different. Ask child which set has more. Student should not need to recount before answering.

1.6 Read and write numerals; Read number words 0 to 10

Because recognition and correct formation of numerals are needed for recording mathematics, children need many tactile experiences to internalize the left-right, up-down configurations which form the ten digits used in our number system. If students consistently reverse digits, have them trace numbers instead of practicing writing them incorrectly. Presenting word forms with numerals and pictures of sets encourages learning of word names. Reading number words should come as a part of games or in other contexts rather than through flash card-type lessons.

A. Have student form numerals with play dough, yarn, or on geoboard. In beginning lessons some students may need a form to follow. Ask student to read numerals that others have made as a part of the lesson.

B. In playing math drill games, student names numerals correctly when drawing cards or rolling a die with digits.

C. In reading for pleasure or other contexts or in playing math drill games, student reads word names correctly.

C. Leader shows student a numeral or a number word and names an action. Child repeats the action the appropriate number of times.

4 kicks

five claps

D. Child reads numbers on calendar, page numbers in books, or numbers in pictures. Student is able to locate the proper page when asked to find a specific number.

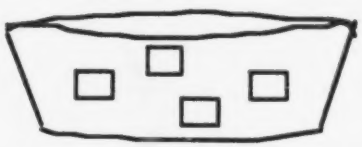
E. Student writes estimate of "how many in the jar", the day's date, or other numbers needed in daily work. Child writes numbers in standard form correctly when they are given orally.

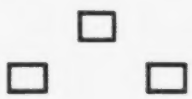
1.7 Count on

Because this is a difficult skill for young children, they will need many experiences before they use counting on consistently as a strategy to determine "how many". Teachers must model the strategy and assist students in using this counting strategy in real-life situations. Before determining that a child has "mastered" the skill, teachers will want to observe a student in many situations over a long period of time.

A. In determining "how many", child may first count a part of the objects and cover them with hand or a cup. After telling how many are covered, the child counts on additional objects.

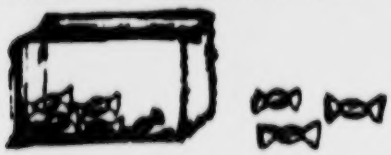
B. Leader makes group of blocks (cubes, children) and tells how many there are. Student counts on as additional blocks are added to the group.



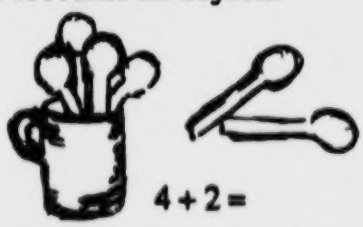


"There are four cubes in the bowl. Count as I drop more in."

C. Child is shown a container with objects, additional "loose" objects, and a number sentence to complete. Ask student to tell how the number sentence and the objects are related. Ask student to find how many in all. Observe if student counts on rather than recounts all objects.



5 + 3 =



4 + 2 =

D. Ask student to continue counting objects, telling the child that you have counted some of the group. For example, "We want to know how many library books our class is turning in today. Here are six. Please finish counting for me." Or, "Susan, Joe, and Tony want pizza. That is three. Raise your hands if you want pizza for lunch. How many should we order?"

E. Student selects a number card and begins to count from that number. *Writing the numbers, starting with numbers other than 1, is a more complex skill that students should work with throughout the year.* Give the student a 2-digit number (for example, 34) and ask the child to name (write) the next 10 numbers.

1.8 Recognize one more/ less/ before/ after/ between

A. Teacher makes a set. *If the objects are available to the student, the activity is at a concrete level. If they are placed on the overhead, they are pictures to the students. Interchange, as appropriate, vocabulary words less and fewer, more and greater.*
Ask child to make a set with one more, or one less, or the same. Ask student to explain how to know that the answer is correct.

B. Student is shown a set. The child tells how many would be in a set with one less or in a set with one more. The student should be able to draw a set with one more or less. A more advanced level would be to give the

student a number card and ask him to make sets with one less or one more. Ask child to verify (explain) the answer.

C. A series of numbers is written on the board. One number is erased from the middle of the series and a line is drawn in its place while student's eyes are closed. The child opens eyes and names the missing number that is between those showing.

D. Student covers numbers on a hundred board at teacher's direction. For example: "Cover 28. Cover the number that is one less than 12. Cover the number between 23 and 25. Cover the number that is one more than 45. Cover the number that comes just before 7. Cover the number that comes just after 81. Tell the numbers that are covered. Explain how were you able to figure out what was covered?"

1.9 Compare/ sequence numerals

A. Given two numbers, ask child to tell which number is greater. Or ask child to circle the larger (greater) number. Ask child which number means fewer (less).

B. Give sequentially numbered cards to members of student teams (one card per child). For example, one team might be given 9, 12, 8, 11, and 10. They race to see which team can order themselves according to the cards first. For advanced groups, use randomly numbered cards for the teams and have them line up from least to greatest. For example, members of one team might be given 36, 14, 83, 57, and 25. Ask a student to explain why they arranged themselves in that order.

C. Student arranges 0 to 9 cards correctly. As student advances, the sequence should use larger numbers. A more difficult level is to have the student order cards that are not in numerical sequence.

D. Student hangs numbers on calendar board correctly or puts pages that have been numbered in their proper order.

E. Given three or four random 1 and 2-digit numbers on a worksheet, student will write their sequence from least to greatest (or greatest to least) correctly.

1.10 Rote count by 1's, 10's, 5's, and 2's; Group to count by 10's, 5's, and 2's

On student profile note how well child can rote count by 1's, 10's, 5's and 2's as well as record the use of the skill as the student counts objects that have been grouped in these ways. Rote counting will be developed through memorization but is much less important than activities that give the sequences meaning. Later writing the counting sequence by 5's or 2's in columns rather than horizontally helps students to see patterns in both the ones and tens places.

A. Child counts while stepping forward (1,2,3,4) and counts back (4,3,2,1) when returning to place. Have child clap when counting forward and snap fingers when counting backwards. Have students form a circle and toss a bean bag or yarn ball, counting as each child catches the ball.

B. Ask child to count by 1's. Note how far the child counts accurately. Later ask student to count again, checking for consistency. Repeat request when appropriate for 10's, 5's, and 2's.

C. If student is counting beyond 30 consistently, help the child enter the counting sequence with a number in the 20's or at another appropriate point in the middle of a decade and continue counting. Ask student to count down from a given number.

D. Given a number, the child continues the counting sequence when playing "Abbracadabra". For example, the teacher says a number and students begin counting from that number. They continue to count until the teacher says "abbracadabra" and names a new number. For example, the teacher says, " Abbracadabra 8". The children continue, "9, 10, 11, etc."

E. Student counts sets of tens. For example, containers with 10 objects, sticks of 10 Unifix cubes, bundles of 10 popsicle sticks, clear sandwich bags of 10 candies, stacks of ten pennies.

F. Student counts sets of ten from pictures or by showing the pattern on a hundred board.

G. Place 5 children in jump ropes or hula hoops or 5 objects in cups. Child counts by 5's. Student may record on the board.

H. Play "Give me five". Students line up, holding out one hand. Child walks down the line, slapping hands and counting by 5's.

I. Make towers of 2 cubes, stacks of 2 blocks, groups of 2 crayons, etc.. Child counts how many by 2's. Student records the counting sequence. Count sets which normally come in 2's. For example, eyes, shoes, knees.

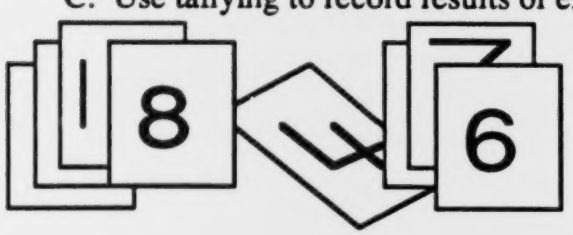
1.11 Count by using tallying

Tallying is a tool used to keep records of events or objects. Before students are asked to tally, they must be taught the process. One strategy is to use a xylophone to help children learn when to make straight marks and when to cross with a slash. "Ding (make a mark), ding (make a mark), ding (make a mark), ding (make a mark), ZIP across all the notes (make the cross slash)."

A. Go on a door knob (hub caps, plants in windows, doors in the classroom) search. Have child tally to determine how many you saw.

B. Use tally marks to keep score in a game or to count how many children want to order certain lunch choices. Have student explain the recording.

C. Use tallying to record results of experiments or to determine how many.



With numbers

Blanks

"How many times did you draw a blank card?"

D. Show child tally marks that teacher has made. Have student tell how many they represent.

1.12 Make reasonable estimates of "how many"

Estimation is developed through activities which build on the students' experiences. Since reasonable estimates are the purpose rather than exact answers, lessons should focus on possible solutions not on one solution. The goal is for the student to continually improve estimation skills; lessons with similar objects should be repeated to encourage students to "build" upon their experiences, moving from guessing to giving responses that have reasons (experiences) to back them up.

A. Show student a group of counters. Hide them to avoid the student counting or recognizing the set if there are fewer than 8. Ask the child how many do you think there are?

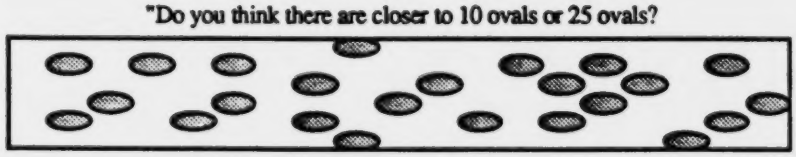
B. How many steps are there from your desk to the door? From the door to the water fountain? From our classroom to the cafeteria? Would it take you more steps to go to the reading table or more steps to go to the teacher's desk?

C. Show student a clear bag with objects. Tell him how many there are. Show him a larger bag with same type of objects. Ask, "How many do you think are in here?"

D. Show student a group of objects. Ask, "Do you think there are enough for each student in the class to have one?" (Or to have two?)

E. Make flash cards with stars, shapes, large dots, etc. Ask student to estimate if the number of objects is closer to one number or another number. Cards should picture an increasing number of objects as student demonstrates good estimation skills with fewer objects.

"Do you think there are closer to 10 ovals or 25 ovals?



F. Place a yellow Cuisenaire rod on the left edge of the desk. Ask student how many rods will it take to go to the end. Add four more rods. Point out to the student how far the five rods go. Ask, "Would you like to change your estimate?"

1.13 Group objects into tens and ones; Record

Because 10 is the number upon which our place value system is based, children must have daily experiences which lead to understandings of 10's and 1's. They need a variety of place value models which can be easily put together and taken apart, showing that one ten and ten ones have the same value. They need experiences with making ten, counting on from ten, and counting to the next ten. They need help in understanding that the symbols they once used for collections of individual objects can now be used for groups of ten objects if you write digits together in a special order. Teachers must help students relate, for example, the 6 tens and 2 ones that the child models to the standard numeral 62. The pattern of writing the digit which represents the "loose ones" on the right and the digit which represents the groups of ten on the left is one which should be developed simultaneously with concrete experiences. Since many first grade children confuse left and right, a place value mat will be helpful. Two sets of 0 to 9 cards used to name the number that is being built is a way to relate sequences of numbers with models when a child adds or removes counters and adjusts the number cards appropriately.

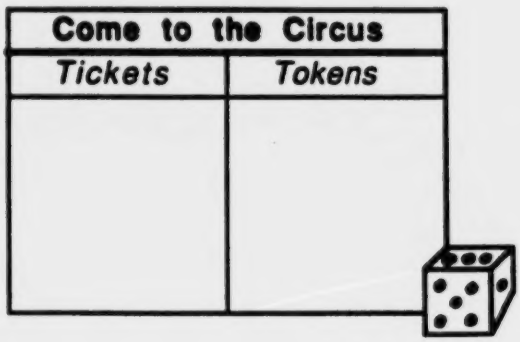
A. Child makes sets of 10 (and identifies any extra ones). String 10 pieces of macaroni. String 10 cherries. Bundle 10 straws. Put 10 beans in a cup. Hook 10 paper clips together. Make paper chains with 10 loops.

B. Given objects, have the student group them into tens and ones. Ask student to count the groups and tell the number of 10's and 1's. Have student cover an area with cubes, organize them into 10's and 1's, and count the groups and loose ones to tell how many. Write the number of 10's and 1's.

C. Combine estimating and finding 10's and 1's by repeating the same lesson several days in a row, changing the number of objects and observing if the child improves the estimates after the third or fourth experience. Place lima beans (less than 100) in a jar. Ask child to estimate how many and then organize the beans into 10's and 1's. Record the tens and ones.

D. Shown a picture of objects, the student circles tens and tells number of 10's and 1's. Students should also give the standard form of the number.

E. Play regrouping games with a partner(s) such as "Come to the Circus". Each student has a gameboard. At each turn the student rolls a die and collects that many tokens (red Unifix cubes). When a student has 10 tokens, they may be traded for one ticket (yellow Unifix cube). The object of the game is to be the first student to win 5 tickets.



1.14 Recognize models; Build 2-digit numbers; Write numerals

A. Child builds a number on a place value mat. The child tells how many 10's and 1's. The teacher or a partner builds a different number and the student tells how many. "Whose number is greater? How could you show one more that the number? Could you show one less?" Writing the standard form is an extension of telling the number of 10's and 1's.

B. Give student a number and ask him to model the number using cubes, tongue depressors, beads, etc. *Use loose objects or those that can be bundled and unbundled easily rather than models such as bean sticks or base 10 materials for developmental lessons.*

C. Teacher builds a number on the overhead. Student models the same number with materials on a place value mat. Student names the number as tens and ones and also as a standard numeral.

D. Partners each build a number less than 70. One student rolls a die. If either student (or both students) have made a number with that many tens, the child earns a point. Play continues until one partner has 10 points.

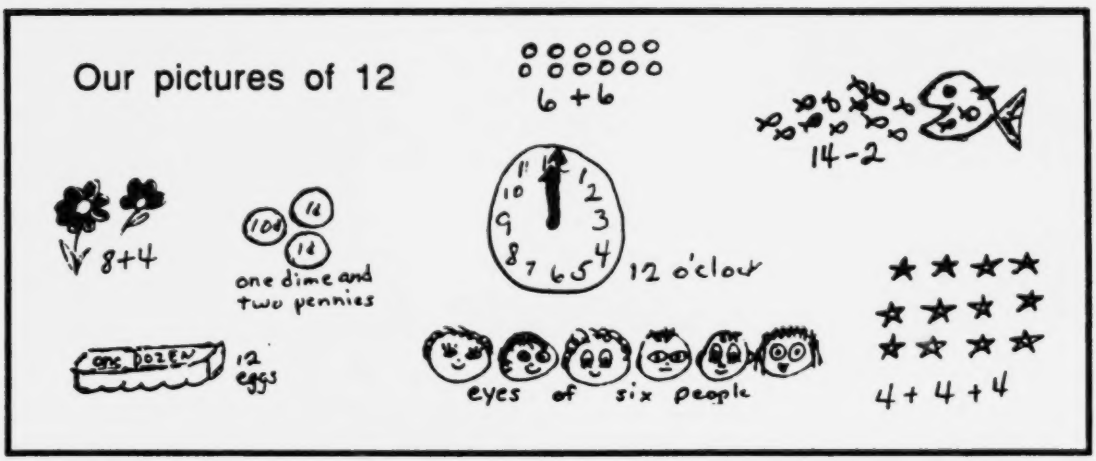
E. Student tells how many 10's and 1's when shown pictures of objects already grouped into sets of tens and ones. The student also names the

number in standard form. *(It is important that students recognize the groups of 10 rather than counting all objects within each group, 1 to 1.)* When given an illustration of 10's and 1's, the student can write the correct numeral.

1.15 Represent numbers in a variety of ways

A. Teacher says "I'm looking for 8. Who can help me find it?" (Legs on 2 chairs, shoes on 4 people, crayons in the box, etc.)

B. Students work in groups to make displays on a large piece of construction paper. Teacher directs them to show 12 in as many ways as they can.

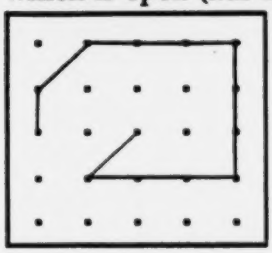
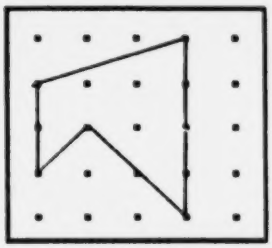
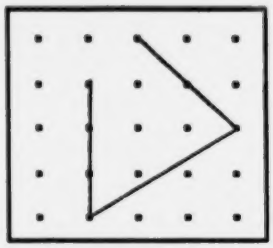


C. Student can explain why 46 is the same as 4 tens and 6 ones. Ask student to also show why 3 tens and 16 ones is the same as 46 or 4 tens and 6 ones.

Competency Goal 2: The learner will identify and use geometric ideas.

2.1 Identify open and closed figures

A. Ask the student to use 4 or 5 straight lines to make a design on the geoboard. Would the design be a fence with no opening to keep an animal in? Have the student make a closed figure (fence) and make one which is open (has a gate).

B. Place yarn on table or overhead. Ask student if the design is an open or closed figure. Change design and repeat the question.

C. Show student pictures of open and closed figures. Ask child to mark all of the closed figures.

2.2 Identify, describe and model plane figures; Recognize in the environment

A. Teacher places triangles, circles, rectangles, and squares on the table. The student is asked to find all of the triangles, etc. and tell about them.

B. Ask student to make a specific polygon, for example, a triangle, on the geoboard. The teacher makes another sample on a second geoboard. Place geoboards side by side to compare and ask, "Are these figures both triangles? How do you know?"

C. Student closes eyes as a shape is placed into the child's hands. Child is asked to describe the shape and identify the figure by touch.

D. Using string, tinker toys, toothpicks, straws and clay, etc. student creates models of squares, triangles, rectangles, diamonds, and circles. Ask student why shapes made with straight lines are easier to model than

circles. Ask student which the student prefers to draw. Why?

E. Given a picture with hidden shapes, student outlines geometric figures.

2.3 Use directional/positional words

Students should experience words which focus on directions or positions such as over, in, under, on, up, down, before, after, between, near, far, first, last, beside, around, above, below, next to, behind, in front of, beneath, right, left, and middle. They need to use themselves as models, toys or counters as models, and finally explain the directional/ positional words in pictures.

A. Teacher reads story such as "Three Billy Goats Gruff" to students. As children retell story, one child acts out each part. (This story uses many comparative words as well as directional terms.)

B. Student uses blocks or cartons to construct a miniature "play ground". Using a teddy bear counter or other toy animal, child will illustrate directional words called out by the teacher.

C. The child participates appropriately in movement songs such as the "Hokey Pokey", Hap Palmer songs, and aerobic dances.

D. Given a picture, the student tells (writes) relationship of figures in the illustration. Teacher will need to model the idea first. "John is walking over the bridge. The duck is under the bridge. The bird is in the tree."

E. Student is able to follow directions by placing self or objects in positions described by the leader. When student can follow one direction consistently, ask the child to follow two then three directions.

F. Student builds a design with pattern blocks behind a folder and then describes the design to partner who tries to reproduce the design from the oral directions.

G. "Fold your paper in half two times. How many boxes will you see when you unfold it? Put your name in the top right corner. Draw a star in the middle of the paper. Trace your hand in the top left hand box. Draw a flower in the top right hand box. In the box below the flower draw a heart. Beside the heart write your favorite number. In the bottom left

box draw a kite. Over the kite print your favorite letter. Draw a line under your hand." *Extend the activity to include other directional/positional words. Repeat the activity frequently until children can complete such tasks in a center using taped directions. These fun directions lead into math related directions which children must understand in setting up problems correctly. For example, write a "+" between the 4 and the 3 or write 16 below 31 and put a "+" to the left of 16.*

2.4 Describe likenesses and differences

A. Two children are chosen to stand before the group. A third child tells 3 ways they are alike and 3 ways they are different.

B. Student chooses several objects (for example, Relationshapes, "junk" boxes, collections of teddy bears) and describes why certain ones go together and how they differ from the others.

C. Give student 2 unrelated objects such as a large blue plastic comb and a blue plastic car. Student then tells as many ways as possible how objects are alike or different.

2.5 Identify and describe solid figures; Recognize in the environment

A. Using geoblocks, student builds a structure. Have the child identify which shapes are the same (i.e. larger or smaller cones or cubes). Ask child to name blocks.

B. Give the student models of cubes, spheres, and cones to handle, compare, and identify. Go on a shapes walk. Ask student to find examples of specific figures.

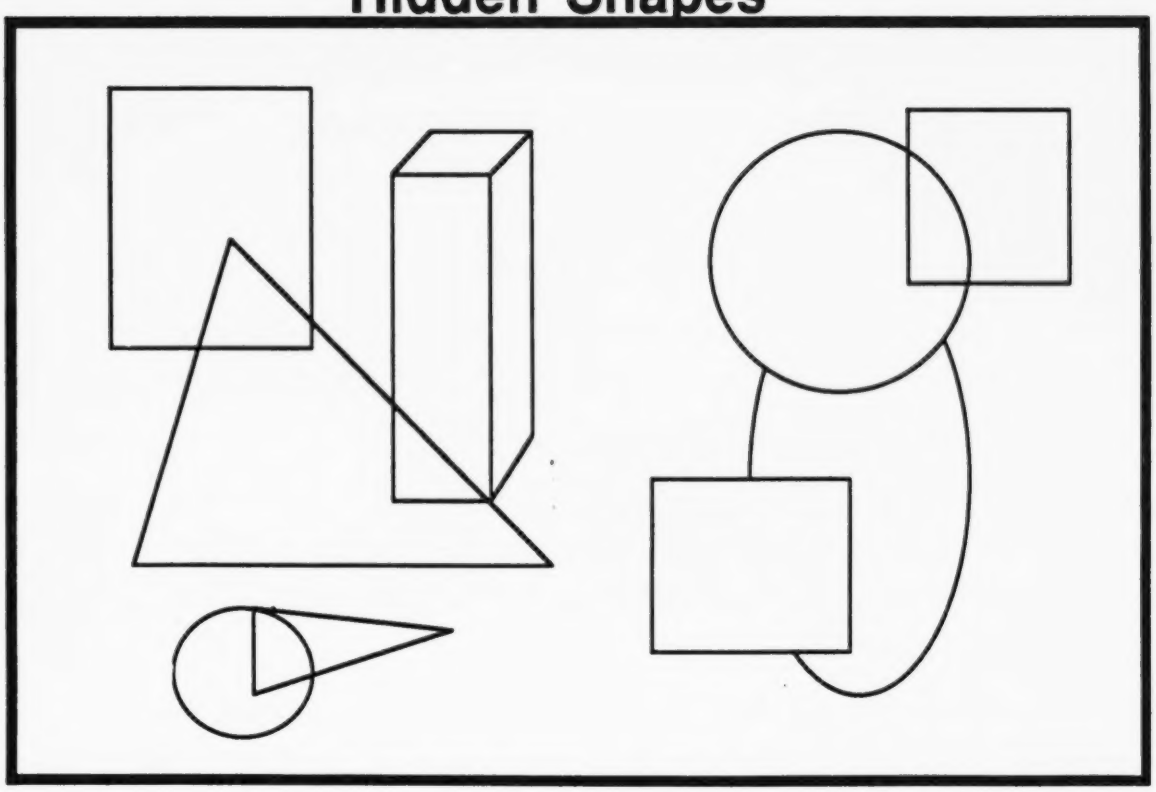
C. Have student make solid figures (boxes, balls, cones) out of clay and straws (or other materials). Have student describe the figures and how they were made. *The use of vocabulary, "sphere" and "cube" for ball and box respectively, should be encouraged but <u>not</u> required. Adults should model new vocabulary to help students learn the terms. Note that not all boxes are cubes.*

D. Place models of many different solids in a box or bag. Child removes without looking the one requested by the leader.

E. Have student bring to class pictures of plane and solid figures; make a book with pictures cut from magazines. Given pictures of solids, ask student to mark those representing a specific figure.

Sample of hidden shapes picture

Hidden Shapes



Competency Goal 3: The learner will demonstrate an understanding of classification, patterns, and seriation.

3.1 Describe objects by their attributes; Compare and order

A. Play "I'm thinking of a student". Give clues to describe a student. Child will identify the person from the clues. Play the same game with children giving clues.

"I'm thinking of a girl." "This girl has on tennis shoes."
"This girl is wearing jeans." "This girl has black hair."
"This girl has on a pink shirt." "This girl has on a belt."

B. Play "20 Questions" to guess what (who) is it. At first have students choose an "it". The teacher asks questions to help children understand the game. Later students can play with teacher or student with student.

"Does it have legs?"
"Is this thing alive?"
"Is it a person?"
"Does it have fur?" etc.

C. Have student smell unknown odors. Ask child to describe how they smell. What do you think you are smelling? If you were going to place these samples on the shelf by the way they smell, how would you arrange them? Why?

D. Give each pair of students an object. Have them describe it, naming as many attributes as possible. Share the list with another group and have the second group identify what is being described.

E. Order six or seven classmates from shortest to tallest.

F. Given a collection of containers, have student order them according to the amount of water (rice, beans, etc.) they will hold. Be certain to have student check the order. Ask student to suggest how the seriation could be checked rather than telling the child what to do.

G. Have student paste 4 pictures in the correct order to show a sequence.

3.2 Sort objects by a given attribute; by more than one attribute; explain sorting rules

Students working together may decide on rules or the teacher may suggest ways to sort. Some children have difficulty forming rules and consistently carrying out their own guidelines, but they are able to follow others' directions.

A. Have children line up according to a rule. See if individuals know where/ when to go. "All children who brought their lunch line up at the door." "Boys get in line." "Children who sit at table 3 may get in line." "Students who brought their library books back may get in line." "All students with tennis shoes may get in line."

B. Direct student to sort the Relationshapes (or other attribute materials) by color, or size, or shape, or any other single attribute. Ask child to tell about the groups.

C. Have student put classroom objects with other things that are just the same. For example, blocks go on the shelf, pencils in the pencil can, coats go on the coat hooks, and Unifix cubes go in the tub.

D. Have student sort books into ones read and ones not read. Choose a book from the latter group to read this week.

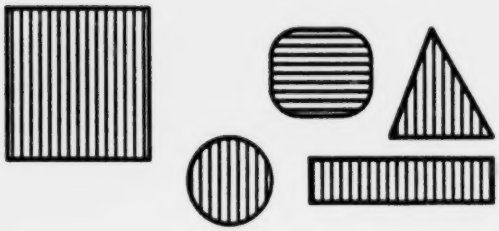
E. Invite different students to the front, placing them into groups one by one (ex. students with jeans and those with short pants). Ask another student, "Which group do you belong in?" Have child explain.

F. Line up students by two criteria: Observe if student knows when to get in line: Students with umbrellas and raincoats go first, then students with umbrellas, then students with raincoats, finally children with neither.

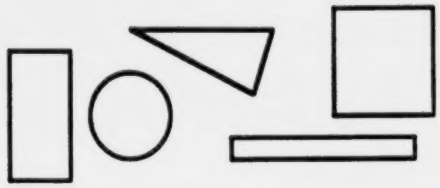
G. Give each student a shape made from construction paper. Ask those with large green circles to stand. Have students with small blue triangles to stand. (Use Hap Palmer song.)

H. Sort capital letters by all straight lines, all curved lines, and both straight and curved lines. Given pictures of animals, mark those that have fur and live in the forest. Ask student to tell about each grouping.

I. Play "Guess My Rule". Teacher or another student sorts objects and asks student to explain the rules that were used.



"These have stripes.



These do not have stripes."

J. Have student make an attribute train. Each "car" must be different from the previous one in two ways. *(It is a simpler task if the directions say "at least two ways". The more difficult task is to make the cars different in "two and only two ways".)* Have student explain the differences as the new pieces are added.

3.3 Sort objects by own rule; explain sorting rule

The importance of language cannot be overlooked. As students are grouping objects, ask them to give reasons for the sorting. When students work in groups, ask them to agree upon a rule(s) before they begin to sort the materials. Having to clarify the criteria for classifying encourages children to use descriptive vocabulary they might not otherwise use.

A. Give student materials (Relationshapes, pasta, keys, buttons, etc.) and ask student to put them into some kinds of groups. Student must be able to explain the groupings.

B. Ask child to organize the books in the reading area. Have student explain how they are grouped.

C. Have student organize items in class store. Explain how things are arranged. Why do certain items go together? Have student sort pictures of foods into groups and then explain how they are classified.

D. Have student cut pictures from magazine. Ask child to classify them and paste them on pages according to the rule the child has made. Have student explain.

3.4 Copy/ continue patterns; Translate into different forms

A thorough understanding of the concept of pattern is important for all students. It is an understanding of patterns that helps to bring order into the child's environment and allows the child to make predictions. Patterns are studied in mathematics, but they are found in all content areas. Read pattern stories and poems, use patterns in art, and look for patterns in science as students are creating patterns in mathematics.

A. Begin simple AB patterns at a concrete level and have student continue them. For example, clap, snap, clap, snap, clap, snap ... green cube, yellow cube, green cube, yellow cube ... stand up, sit down, stand up, sit down.... Ask child to explain what the pattern is. Ask child to show the same pattern with different actions. Ask child to describe the patterns using letters.

B. Observe if child participates correctly in pattern songs and dances.

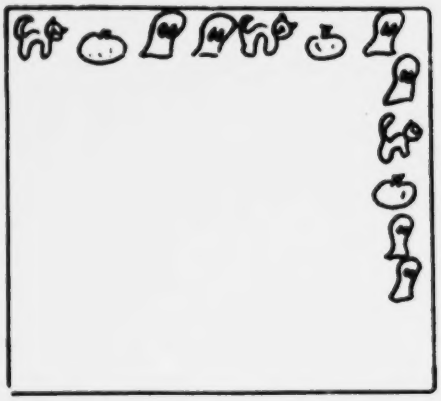
C. Provide templates, stamps or stickers for child to use in copying a pattern. Ask student to explain the pattern you have created and to continue the pattern. Use adding machine tape and have student repeat pattern to create bulletin board borders.

D. Have student continue more complex patterns. Ask student to explain the pattern unit. Ask student to make the same pattern with different materials. For example, the pattern below is ABB. It might be translated into "hands up, clap, clap".

E. Given pictures of objects arranged in linear patterns, ask child to identify what comes next. Ask student to make a similar pattern with different materials.

F. Have students choose their favorite holiday or season (each student may be different). Give them a basic pattern and have them each create a

holiday place mat with the same pattern but with figures that relate to their special day. For example, tell student the pattern is ABCCABCCABCCABCC.



G. Have students discover patterns with 2's, 5's, and 10's on hundred boards up to 50. Use Unifix cubes or other markers to cover the numbers. Record these patterns by coloring 100 charts to show each sequence. At an abstract level, write the sequence.

3.5 Create patterns with actions/ words/ objects

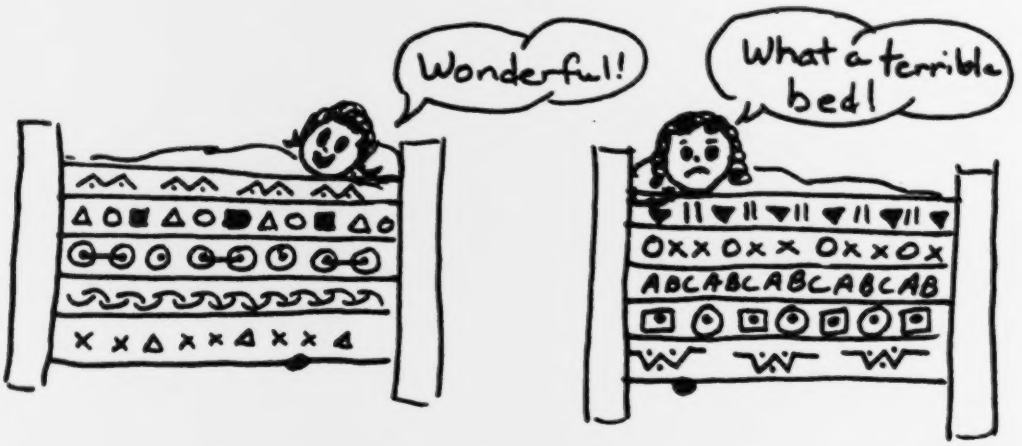
A. Have student lead the class in an action pattern. The child makes up an action pattern or arranges fellow students into a pattern. Have student explain the pattern.

B. Given Unifix cubes, pattern blocks, or other interesting materials, have student make a pattern. Ask child to explain the pattern.

C. Provide stamps, templates, crayons, paper shapes, etc. Have student to make borders on place mats or for picture frames. Create wrapping paper. Ask student to explain the pattern.

D. Read to students the story of the "Princess and the Pea". Have students design pattern strips to be the mattresses for the girls in the story. Ask children to dictate sentences about the story. Give a group the assignment to sequence the sentences correctly and then make a bulletin board using

their "mattresses" on the girls' beds.



3.6 Find and correct errors in patterns

A. Show student a simple pattern with pattern blocks or Unifix cubes that has an error. Ask student to tell about your pattern and notice if the error is discovered.

B. Using objects from the classroom, create patterns which have errors. Ask student to explain and correct the mistake.

3.7 Identify patterns in the environment

A. Child identifies patterns in children's clothing or in wallpaper samples.

B. Take students on pattern walks. Have student tell their partners about the patterns they see around the school such as tiles, bricks, blinds, etc.

C. Have student make crayon rubbings to show patterns (tennis shoe bottoms, bricks, leaves from same tree, tree bark, etc.).

D. Have student identify environmental patterns by finding pictures in magazines to illustrate them. Ask student to make a display and explain it. Such displays might include pictures of the four seasons or pictures showing the day-night cycle or pictures showing the usual meal cycle or pictures of a collection of floor tiles or a series of window patterns in large buildings.

Competency Goal 4: The learner will exhibit skill in using units of measurement.

4.1 Compare objects/ use appropriate vocabulary

Students should have many opportunities to experience comparisons and hear and use the appropriate vocabulary. A rod that is "taller" in one situation may be "shorter" in another. Vocabulary should include positive, comparative and superlative forms of words such as tall, short, long, small, large, big, little, old, young, new, thick, thin, heavy, and light. More and less should be explored as geometric terms as well as numerical ideas.

A. Ask 2 students to stand. Have another child tell who is taller (shorter)? Direct one of the students to choose a friend and exchange places. Now who is taller (shorter)? Make other comparisons with people and familiar objects in the classroom.

B. Set up a table with six "compare" stations - two each of weight, length, and capacity. Have student compare objects at each station, explaining the comparisons. As student becomes confident in the measurement process, have child record the results of the comparisons.

C. Use a balance to find the heavier object. (blocks, books, toys, etc.) Have student explain what happens to the balance when one object is much heavier than another. How would a picture look with a heavy can in one balance pan and three cubes in the other balance pan?

D. Give children stacks of one color of Unifix cubes (different lengths). The children must find other students with a stack the same size as theirs. (Use string, ribbons, or construction paper strips instead of cubes.) Child finds 2 more people -one with a shorter stack and one with a longer stack.

E. Give student two objects to describe and compare. Ask child to tell how to decide which is taller or shorter or heavier (etc.).

F. Have student compare and make a chart. Title the chart appropriately: "We compare our shoes". As students measure, each child would record the other's name in the appropriate column.

Comparing our Shoes	
Shorter than my shoe	Longer than my shoe

G. Given pictures of 2 objects, ask student to make appropriate comparisons and explain the reasons. "Which person is older?" Which tree is taller?" Which is smaller?" Use pictures of 3 objects for more advanced students.

Workbook pages which picture comparisons are useful in testing if a child understands the vocabulary; however, they are not helpful in teaching the child the meaning of the ideas.

4.2 Use non-standard units to measure length, weight, and capacity

Before students use standard measurement units they should understand the process of comparing and of measuring. Mass (weight) and capacity frequently fool adults as well as children when visual clues alone are the basis for decisions. Students need to know why standard units are necessary. They must understand what happens to a balance when one side is heavier, how a ruler is made, and what type of measurements are appropriate (i.e. a piece of string is not a good tool for measuring capacity or weight, but it is good for determining length). These mathematical explorations should come before the introduction of standard measurements.

A. Using Cuisenaire rods or Unifix cubes or straws, measure the length of the teacher's desk, a tall book, and the bookcase. Use paper clips to measure shorter objects. If you use rods of a different color or larger paper clips, would the answer be the same?

B. Determine the weight of classroom objects using counters as nonstandard units and a balance. Estimate and then check to see how many counters balance the object.

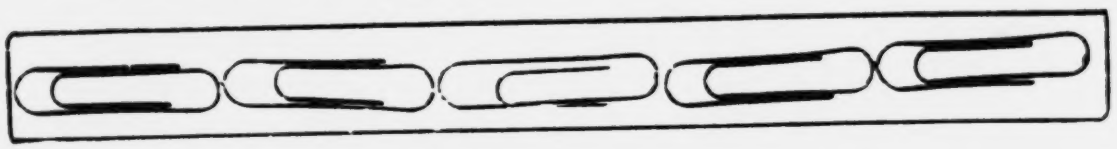
C. Blindfold the student. Give the child 2 objects to estimate which is heavier. Check with a balance.

D. Estimate and then measure: how many scoops (sand, rice, beans, water) are needed to fill the coffee cup; how many scoops are needed to fill the glass; how many scoops are needed to fill the bowl. Which container holds the most? Which holds the least? *A measurement center, which is set up with materials for linear measurements for two weeks, then materials for weighing for a time, and then containers and materials for exploring capacity for a couple of weeks,*

will afford students "hands on" experiences without burdening teachers with the need to gather materials for separate lessons. The same centers can accommodate students who are more advanced and ready for exploring standard units.

E. Have students cut 10 equal units (these need not be standard). After using the units loose, have students make a "tape measure" by taping the units together (a ribbon backing makes assembling the tape easier). Ask student, "Do you get the same answer when you measure your book with your units loose as you get when you use your measuring tape? Is it easier to measure with your loose units or the measuring tape? " *Have students cut their units from two different colors of construction paper. Be certain to spend time discussing how two children measuring the same object could both have correct but different measurements because their units are different. This discussion will arise as the tapes are being made because students will need different lengths of ribbon upon which to attach their units. The units might be "personalized", that is, the length of Sam's little finger or Jana's foot. Is there anything that we could do to make measuring with the tapes faster? Let students suggest numbering the units.*

F. Given pictures of objects, student will measure with paper clips. *Students can make rigid paper clip rulers by gluing the paper clips to tag board with white glue that dries clear. Ask students to explain where they might put numbers on their rulers to make measuring faster.*



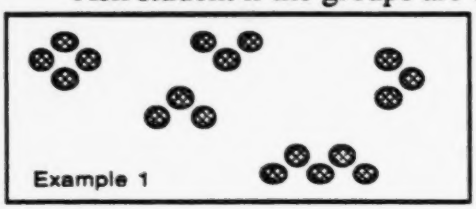
G. Give student 3 objects to measure (for example, a book, their friend, length of room). Ask child to decide which non-standard units would be best to use.

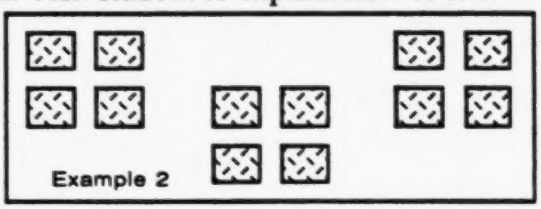
4.3 Identify equal and unequal parts

Many children appear to master fraction concepts during primary years only to demonstrate a very poor understanding when the ideas are applied in upper elementary and middle grades. More important that recognizing and writing the notation for one-half or one-fourth in early grades is the internalization of basic fraction concepts. Being able to recognize equal and unequal parts is a fundamental understanding that rarely receives much attention.While most first grade children recognize and can model "halves", many fail to realize that "halves" are equal parts.

A. Divide shapes (paper, pop tarts or other edible models) into parts. Ask student if all parts are the same size. Explain how you know.

B. Use Unifix cubes (paper clips, counters, cereal, etc.) to make groups. Ask student if the groups are equal. Ask student to explain how to tell.





C. Use ten counters and divide them into various groups. Ask student if the groups are the same (equal).

D. Show students regions which have been divided into parts. Ask child if the areas are equal. (Equality of area, not number of parts, is important.)

E. Ask student how many pieces there will be if something is divided into halves. Are there any special qualities that we could notice about the halves? (They will be equal parts.)

F. Given pictures or a worksheet, student will identify regions or sets that have been divided into equal (or unequal) parts.

4.4 Use time related words in daily vocabulary

A. Given a set of events, ask student to tell when these might take place. For example, brush your teeth, go to a movie, go to the school cafeteria,wear your pajamas, go the swimming pool, leave school to go home. Ask student to relate two events, observing if the child uses words such as yesterday, today, earlier, later, before, after, next, morning, afternoon, night, day before.

B. Give student a series of pictures (cartoons) that might have several logical orders. Ask child to put them in order and tell a story.

C. Observe if student uses appropriate time words in relation to the calendar and to the daily schedule. For example, "After we come back from lunch, we will go outside for P.E." "Next week is my birthday." "Yesterday was April 12."

4.5 Name and order days of the week; name months of the year

A. Ask child to name days of the week. A more difficult task is to begin mid-week and name the days. How many days are there in a week?

B. Teacher writes days of week on board. Student reads them and closes eyes as teacher erases one word. Student opens eyes and tells what is missing.

C. Ask student what day comes before Wednesday? Before Friday? After Thursday? Today is Thursday; what day was yesterday?

D. Give cards with one day of the week written on them to seven children. Have another student arrange them in order. A more difficult task is to give a student the days of the week in a list and ask the child to put them in order by numbering them correctly.

E. Ask student to name months of the year. How many are there? What is your favorite month? Why?

4.6 Use information on a calendar

Like hundred boards, calendars are useful in helping children learn about number sequences and patterns. When working with the calendar, talk with students about the numbers - reading the numerals, what numbers come before and after a date, what is between given dates, etc. Aside from the information they give us about the current day, week, and month, calendars also help children begin to form concepts of time. How long a day or a week is begins to have meaning in relation to what can happen in these periods of time.

A. Tell child today's date. Ask the student to find it on the calendar. What is tomorrow's date? What day was yesterday? What will the date be one week from today ? What year is this? Ask student what month is this. What was last month? Do you know what next month will be?
B. Have student locate a certain date on the calendar. Ask the child on which day of the week is that date. Is the date a school day or a weekend day?
C. Ask student in which month(s) these events come: Halloween, Valentine's Day, summer vacation, Christmas, your birthday.
D. Post three sets of days of the week cards so that students see the pattern of days. Ask student how many weeks are posted.
E. Ask student to name the month for a given calendar page. How many days are in this month? What is the date of the first Monday?

4.7 Tell time to hour (face and digital clocks)

Telling time is difficult for most children. Like learning about money, learning about time should be lessons throughout the year, not one isolated unit. Having an idea of the duration of different units of time comes through multiple experiences and with age. Some activities focus on duration (In one minute how far can you walk around the playground?) and other activities focus on the mechanics of reading a clock. One useful idea is to make circular clocks with only one hand. Using the short hand only, students learn the direction in which the hand moves. They can practice "before" and "after" numbers and learn the position of the short hand for o'clock. Activities which focus of reading the clock should include pictures of times on circular clocks which match those on digital clocks.

A. Ask student which hand is the hour hand. Where does the long hand point when the time is o'clock?

B. Ask student to name some things that are usually completed in one hour or less (for example, eat lunch). Name some things that take more than an hour (for example, going to a movie). Extension: Ask student to tell something that could be done in about one minute.

C. Have student identify clocks telling time to the hour. What times do they show? Which way do the hands move?

D. Have students place hands on clock to show specified hours. What would the digital clocks say at these hours?

E. When shown pictures of clocks, student will write the correct o'clock and the corresponding digital time.

4.8 Identify and give the values of pennies, nickels, and dimes

Students may not be equally successful with activities with real coins and the same activities with pictures of money. Actual coins should be used whenever possible for matching fronts to backs, learning values of individual coins, evaluating sets, and making change. Teachers must also insure that students can recognize the pictures and blackline drawings of the coins if students are to be tested in this manner. Since students will use real coins in daily living and eventually move to descriptions of sets (How much money is three dimes, four pennies, and one nickel?), the main reason for spending time with pictures is for pencil and paper testing.

A. Ask student to identify coins (both heads and tails) and state their values. Which coin is worth the most? Which is worth the least?

B. Ask students to identify coins through riddles:

"On one side of me there is a monument.
On the other side is Abraham Lincoln.
I am copper colored.
I am worth one cent. Who am I?"

C. Have student show the value of a dime with only pennies, with only nickels, with a nickel and pennies.

D. Play coin bingo or coin concentration, matching values of small sets with pictures of the coins (or real coins glued to cards).

E. Ask student to recognize coins from pictures and line drawings. Match heads and tails and tell the values.

4.9 Identify coins needed to buy items; make different sets with same value

A. Mark price tags with the following values: (8¢) (10¢) (12¢) (15¢) Ask student to show what coins are needed to buy each item. Increase difficulty of task by marking items with higher amounts: (16¢) (23¢)

B. Place amounts of money in Ziploc bags. Have student match bags with items marked with those amounts. *Vary amounts of money according to students' performance. Students should work with larger sets of coins only after they have had many place value and counting on experiences.*

C. Given pictures of coins, have students circle those needed to buy items.

D. Have student use a calculator to find the value of sets of coins. Use calculator to figure the total cost of two or three items. Show the coins needed for that purchase.

E. Leader calls out (or writes) an amount of money (17¢). Student makes that amount in two different ways.

F. Ask student to use coins, play money, pictures of coins, or coin stamps to make a display: "Three Possible Ways to Make 20¢" (or other appropriate amounts).

H. Ask two students to make 25¢ in as many different ways as they can. Have students organize them in some way to be sure no set is repeated. Ask the two students to make a chart to show what they made. (The activity could serve as a challenge for an advanced student working alone.)

Competency Goal 5: The learner will use mathematical reasoning and apply problem solving strategies.

5.1 Use calculator to solve problems

A. Ask student to read the digits on the calculator display. Ask child to make the calculator display given numbers. Have student clear display. Tell student number sentence to solve. Note if student can correctly input addition and subtraction problems.

B. Show child containers with 34 objects in one and 28 objects in the other. *(The exact numbers do not matter.) The activity is to evaluate if student uses the calculator to solve addition and subtraction problems, correctly inputting the information.* Ask child to tell how many in all using the calculator. When student has the correct solution, put all counters together to model what child has done on the calculator. Have student clear the display. Review the total number of objects and then remove 12. Ask student to use the calculator to determine how many are left.

C. Ask student to use calculator to solve word problems in textbook. *Be certain to use problems where the calculator is beneficial. Some problems are easier solved in your head, and students should recognize this from the beginning. Problems with 2-digit numbers and several addends are appropriate since the calculator becomes a tool for making the work easier.*

D. Use calculator to find sums/ differences in classroom situations. For example, if students count the number of M & M's in their individual bags (or cubes or beans, etc.), how many are there in the entire class?

5.2 Use visual memory; solve spatial visualization puzzles; copy simple designs

A. Teacher places five (or an appropriate number) objects on the table. Student looks at the collection for a few seconds, and then closes eyes as the teacher removes an object. Ask student what is missing.

B. Place objects (pattern blocks or unique objects) in a row. Allow student to study for a few seconds. When student's eyes are closed, scramble the objects and direct student to put them back in the correct order.

C. Fill in an outline with pattern block pieces. (Or use Relationshapes or tangram puzzles.) Given a pattern block puzzle outline, have student complete the puzzle in more than one way. Solve jigsaw puzzles. Have student create a puzzle for others to solve by cutting up old greeting cards into puzzle pieces. Store individual puzzles in sandwich bags.

D. Ask student to copy a geoboard design onto another geoboard. Have student record a design from a geoboard onto dot paper. Another task is to copy a geoboard design that has been drawn on dot paper. (Designs need not be regular figures.)

E. Give student a variety of pattern blocks. Teacher makes a design with four (or an appropriate number) blocks. Child looks at design before teacher covers it up. Ask student to choose the correct blocks and make the design.

F. Given a small sample of wallpaper, have student copy that portion, extend the design, and color an entire sheet.

G. Show how your partner's design would look in a mirror (reflection).

H. Leader builds a design with pattern blocks or other materials. Student copies the design. Important: seat partners side by side. Student studies simple drawing or shapes on overhead then reproduces the drawing from memory.

I. Student creates pattern blocks or Unifix design. Then the student copies the design by gluing construction paper shapes (pre-cut to match pattern blocks or cubes in color and shape) to a background.

5.3 Estimate/ suggest reasonable solutions to problems

A. Ask student if there would be enough pieces of candy in a bag to give all first graders in the room three pieces.

B. Tell student you want six children to use the pattern blocks in one box. Ask child to show you approximately how many each person would have.

C. Ask child to suggest how many name tags you might need to cut out for a "Back to School Night" or a "Family Problem Solving Night". If the name tags are a certain size (show sample), how many could be cut from a piece of construction paper (show the paper).

5.4 Use models or "act it out" to solve problems

A. State a simple problem. Ask student to act it out as part of the group. Give student manipulatives and ask student to model the story.

B. Using a workmat (storymat), teacher (or another student) tells stories and asks student to model them. Observe if student can model problems accurately. Ask student to make up a story for others to model.

C. Ask student to solve problem, directing other students or using objects to act out the solution. For example: "There were some teddy bears in line for honey. The first one was sitting on a stump. The next two were telling stories to each other. The last one was reading a book. How many pots of honey does Mr. Bear need so that each teddy bear will have one?"

D. Have materials available. Ask student to solve a problem such as this: "Holly has three nickels. At the gum machine she discovers that she can get foursmall pieces of gum for one nickel. How many pieces of gum could she buy with her money?"

5.5 Use drawings/ diagrams to solve problems

A. Ask student to interpret several sentences (paragraph) with a drawing. Students need to be able to visualize what is being described and to record the information in some manner. Illustrating stories that are read to the class (stories which have no pictures) encourages creativity, spatial visualization, and the use of drawings to clarify situations.

B. Ask child to show the most direct route from the playground to Mr. McGregor's on a simple map. Ask the child to suggest an alternate route if the road is washed out.

C. Tell child that there are three flavors of ice cream available. How many different two-scoop cones can be made? Allow student to justify if chocolate on top of vanilla is the same (or is different) as vanilla on top of chocolate.

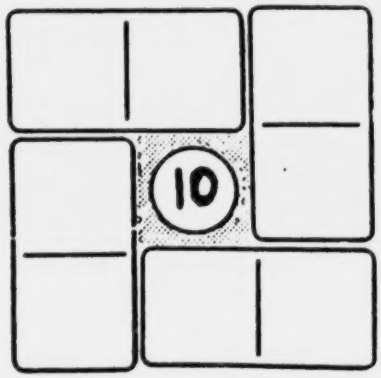
5.6 Guess and check to solve problems

A. Have students play Nim-type games and challenge them to find a way to always win. For example, there are nine red apples (red Unifix cubes) and one green apple (a green Unifix cube). The object of the game is to collect one or two apples at a turn, but to not be the person who gets the green apple. The person getting the green apple loses.

B. Give student a problem in which the answer could be several combinations. Show student six toys with imaginary weights indicated. Judy packed three toys into a box. The package weighed 15 pounds. What three toys did she pack?

C. Place dominoes on the "donut" so that all sides equal 10 (or other appropriate numbers).

D. Make three domino rows with the same sums on each.



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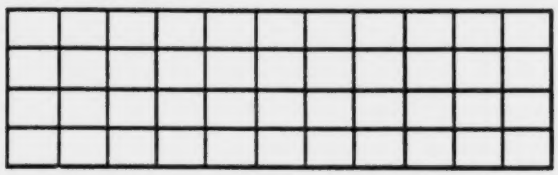
Competency Goal 6: The learner will demonstrate an understanding of graphing, simple probability and statistics.

6.1 Gather, organize, and display information as a group activity

We live in an information world. Sometimes the information we need is not readily observable and data must be collected. What is the question we need to ask to get the information we want? How can we collect the information? How should we organize and display the information we have collected in order to best examine the relationships within the data? Gathering information, organizing it, and creating appropriate displays are tasks first accomplished as part of the group and later mastered individually. Students' mastery of basic graphing skills should be accompanied by understandings of how information can be useful. Teachers must encourage children to think beyond the obvious, to make observations and predictions, and to continually ask questions. Graphing should not be taught as a unit but interwoven into the curriculum throughout the year. It offers many opportunities for integrating math lessons with other content areas.

A. Have student scoop up a handful of pattern blocks. Ask child to sort the blocks and place them on a grid. Ask child to explain about his graph. "Would another student doing the same activity have the same blocks? If you repeated this activity would you have the same blocks? Explain."

B. Observe if student records his opinion by placing a marker (or the object) correctly on the graph. *For graphing activities throughout the year, cubes made from milk cartons can be used for individuals to mark their choices. Write the child's name on one side of a cube, attach a snapshot on another face of the cube, write "yes" on another side, and "no" on a fourth side. Graphing mats made with masking tape and plastic trash bags, painters' drop cloths, old window shades, or oil cloth make graphing lessons fun and easy. Desk top grids that are laminated are also helpful in graphing activities in which Unifix cubes or other smaller objects are used as markers.*



Graph opinions and preferences, data collected by students, classroom information (attendance, weather), and the results of simple experiments.

C. In a group compare the length of each other's names. "Whose name is longest? Are there any students whose names are the same lengths? Is anyone's name shorter than the others? What other questions could be answered based on the graph?"

S	A	M	U	E	L			
K	A	T	I	E				
W	I	L	L	I	A	M		

D. Have students to choose a question they would like to answer. For example, "How do most students come to school?" Have them decide how they will gather and record the information. Ask the students to make a chart or graph to show what they found out. "What categories need to go on the display? Would the results of the survey vary if the information was gathered on a rainy day instead of a sunny day?"

E. Many items are packaged by weight. Does this mean that there are the same number of things in each package? Have students use a balance to compare unopened packages and then count and graph the number of items in each package (individual packs of mixed fruits, M & M's, homemade cereal mixtures, peanuts in the shells, bags of wrapped candies, etc.)

6.2 Answer questions about charts and graphs

A. Ask student to explain information on a graph the child helped to create. In addition to obvious questions about the most, least, same, and how many, teachers should include predictive and opinion questions.

B. Student will correctly answer questions about simple graphs in first grade textbook.

6.3 Make predictions based on experiences

A. Student will give logical responses to questions related to real life situations. For example, "The weather man is predicting that it will rain tomorrow. Should you take an umbrella or wear a raincoat to school?"

B. "If I have four red Unifix cubes in a brown bag and take one out, what color cube could I pull out? If I add one yellow cube to the bag and then pull out a cube, what color am I most likely to get?"

C. "If I roll a regular die, am I likely to roll six more often than any other number since that is my favorite number?" Ask the student to predict what might happen when the die is rolled 20 times. Have the student experiment and record the results.

7.1 Model concept of addition

A. Have objects on the table. Ask the student to explain what it means to add and to show you what might happen to these things if some were added. *If a student understands the concept of addition, the size of the sets or the number of groups should not matter. The purpose is to observe if a child understands the concept of addition, not the recording of the process. Many children understand the concept of adding three groups of things together, for example, but cannot write a correct answer for 4 + 3 + 2. What the student does not understand is how the symbols relate to the groups and what has happened to them.*

B. Observe if student uses terms such as add, total, sum, and equals in (math) conversations.

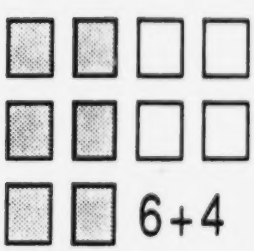
C. Note if student uses manipulatives (models) to solve addition problems.

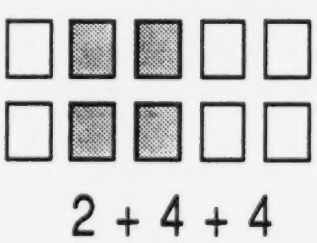
7.2 Explore possible combinations for numbers to 10

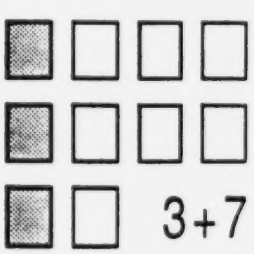
A. Give student ten counters. Ask child to show three different ways to put them into two sets. "What number sentences are you showing? Are there any other ways to arrange them?"

B. Ask student to show all of the ways to make 10. Have the child to arrange the models in some organized fashion and to explain what has been done.

The activity should be repeated many times as a math lesson using many different materials. For example, use two colors of pattern blocks or Unifix cubes, use toothpicks (position would determine addends), glue paper squares in two colors or two colors of cereal, etc.







C. Ask student to answer the questions using counters if needed. "If you have eight counters, you would need how many more to make 10?" "If you have five counters, how many more would you need to have 10?"

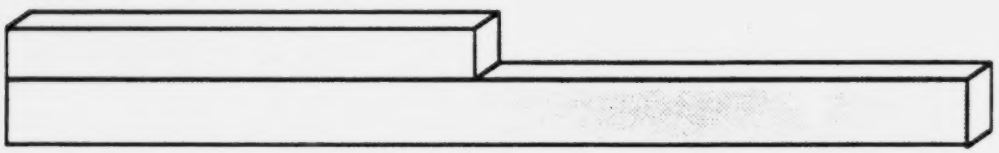
D. Ask student to write all the number sentences whose sum is 10.

7.3 Model concept of subtraction as take-away; as comparison

A. Give student ten counters. Ask child to remove some and tell how many are left. Repeat with other numbers. Have child tell a story about what is taking place.

B. Ask student to model with counters stories such as this: "I have seven and you take three away. How many will I have left?"

C. Have two sets of Cuisenaire rods available. Give student a brown Cuisenaire rod and a yellow rod. Place the yellow rod on top of the brown. Ask the child what rod would fill the space so that there is a "two-car train" the same length as the brown rod. Have child explain. Repeat with other rods.



D. Show student a counting mat with 10 spots. Cover 6 with counters. Ask student how many more are need to make 10. Repeat, covering 8 spots.

E. Ask student to use counters to model the story you are telling. "Sam has 6 cars and Joe has 9. How many more does Joe have than Sam?" or "Jewel has 8 cubes and you have 5. How many more do you need so that you will have the same number?" *Note that these are two different ideas. Children need to experience both kinds of comparative subtraction with active lessons and the teacher writing the number sentences which relate to the situations before students are asked to deal with them in textbook word problems.*

7.4 Create and solve problems using addition/ subtraction

A. Using a story board and counters, ask student to tell a story and use the counters to explain.

B. Give student two numbers. Ask child to make up a story using the numbers and tell the solution. (The student does not need to be able to solve the problem abstractly.)

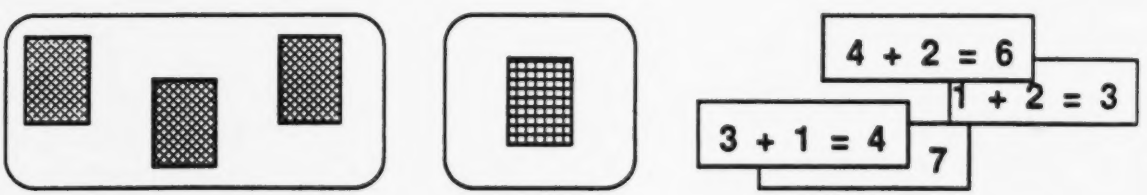
C. Given a number sentence such as 4 + 3 or 7 - 2, ask student to tell a story and find the solution.

D. Give the student a cartoon picture or one from a magazine. Have the child write (tell) a story that has a problem which could be solved by adding or subtracting.

7.5 Relate addition/ subtraction process to symbolic notation/ write equations

A. Given counters and an addition sentence such as 4 + 5, ask student to explain and show what it means. Repeat the process with 3 + 5 + 4.

B. Model a simple addition or subtraction problem. Have the student select the correct number sentence.



C. Model a simple addition or subtraction problem. Have the student write the correct number sentence.

D. Observe if student can record correctly equations in playing games with dice or cards.

7.6 Use counting strategies to find sums and differences

A. Show student that you have seven Unifix cubes in the bag. Ask the child to continue counting as you add two more cubes. Repeat.

B. Show student pictures which encourage counting on as an efficient method of finding the sum. Ask student to explain the pictures and tell how many. *"THREE...four, five"*

C. Show student a number sentence and ask the child to solve it. Have counters available. Observe if child makes sets and recounts both sets or counts on (either using objects or the symbols alone).

7.7 Memorize easy addition facts

7.8 Memorize easy subtraction facts

The focus on memorizing addition and subtraction facts and giving up counters (and fingers) early in the year and the great amount of time spent with pencil and paper drills has brought about two major concerns for mathematics educators. First, both teachers and students are frustrated by the apparent lack of long term retention resulting from this memory work. Secondly, because of the amount of time devoted to these drills, mathematics in primary grades tends to be narrow in scope. There does not seem to be time for exploratory lessons, problem solving activities, and "hands-on" experiences in other strands of mathematics.While it has traditionally been the primary goal of first and second grade mathematics classes that children memorize the number facts for addition and subtraction, this task continues to prove difficult for students. Teachers have responded by beginning very early in the year with flash cards and games for drill and practice. Yet third, fourth, and fifth grade teachers report that a large percentage of their students do not know these facts nor are they able to use the facts to solve problems. The ultimate goal of memorizing addition and subtraction facts will remain an appropriate one, but only when educators acknowledge and attend to the influence of each individual's maturation and each child's concept of number and the symbols which represent these amounts. This means that children must spend a great deal of time comparing groups, putting objects together and taking them apart, and talking about these processes as they occur in their daily lives. The symbols must be used to record what is happening and not be a world apart from objects

and processes. In the book Developing Number Concepts Using Unifix Cubes, Kathy Richardson compares learning about chocolate with the manner in which we expect children to learn number concepts. "No one believes that learning to read, write, and spell the word chocolate is synonymous with experiencing chocolate; but many do believe that knowing how to read and write 3 + 4 = 7 is synonymous with understanding the number concepts represented by these symbols." (p.77) Attending to a child's level of maturity also means that different students will master memorizing number facts at different rates. Students should not be rushed into drills with symbols until they begin to demonstrate a knowledge of facts through a concept of the quantities the numbers represent. Efficient use of strategies (such as doubles, counting on, commutative and doubles plus 1) are appropriate goals for hands-on experiences and lead to memorization based on understanding.

A. Ask child to respond to "easy" addition and subtraction facts when they are called out to the student. "Memorize" means that student does not need to use counters or fingers and can respond quickly. Rather than penalize a student for facts the child does not know, keep a list of the facts a student has memorized. *The chart on the following page shows easier addition facts. "Easy" facts are those which most children learn first because of counting strategies (a number plus one or a number plus 2) or rote phrases (doubles). Easy is a relative term, however, and should not limit nor dictate exactly what a student learns. Some teachers will emphasize "fact families", especially those to 10.*

B. Ask a student to respond orally to the facts when shown in a written form such as flash cards.

C. Ask student to write answers to number facts when presented on a worksheet. (Time limits penalize students whose motor skills are not conducive to rapid writing. The best evaluation of memorization is oral.)

D. Given real life situations involving addition or subtraction of easy facts, student will respond without counting.

7.9 Model 3 single-digit addition; record

A. Student pulls a handful of cubes from a bag that contains three colors of Unifix cubes. Child records on a chart how many of each color and

writes the total. A more advanced level has the child to write the number sentence. *It is extremely important to relate the actions of adding three groups to the manner in which the action is recorded. Many children will add 3 + 4 + 2 successfully when it is written horizontally; but they will write 63 or 36 or 72 or 27 as the answer when the problem is written vertically, adding two of the addends and "bringing down" the third. To introduce students to addition with three or more addends at an abstract level rather than concretely because the students are successful with problems with two addends is to make the assumption (frequently false) that children intuitively understand the new symbolic notation.*

B. Given a problem with three addends, student will model with counters. More advanced students will solve the problems, especially if the addends are small, without counters.

C. Have student interview ten to twelve students, asking their preferences on a question with three possible responses. (For example, at snack time today do you want orange juice, apple juice, or lemonade?) Have student explain results of the survey and then write an appropriate number sentence to account for all choices.

7.10 Model 10 more/ less

When students are using manipulatives or hundred boards to show ten more or ten less. they are developing a sense of numberness that leads to success with mental math. estimation, and resolution of written problems. First experiences with two-digit operations should be with adding or subtracting ten (20 + 10, 32 + 10, 40 - 10, 53 - 10).

A. Student builds a given number and adds to or removes counters to show ten more or less.

B. Given a model, student demonstrates how to alter the model to show ten more or less than the number. Have student explain.

C. On a hundred board student marks a given number and then identifies ten more or ten less. A more advanced level student will tell or write the number meaning ten more or less than a given number without models.

7.11 Model 2-digit addition and subtraction with multiples of 10

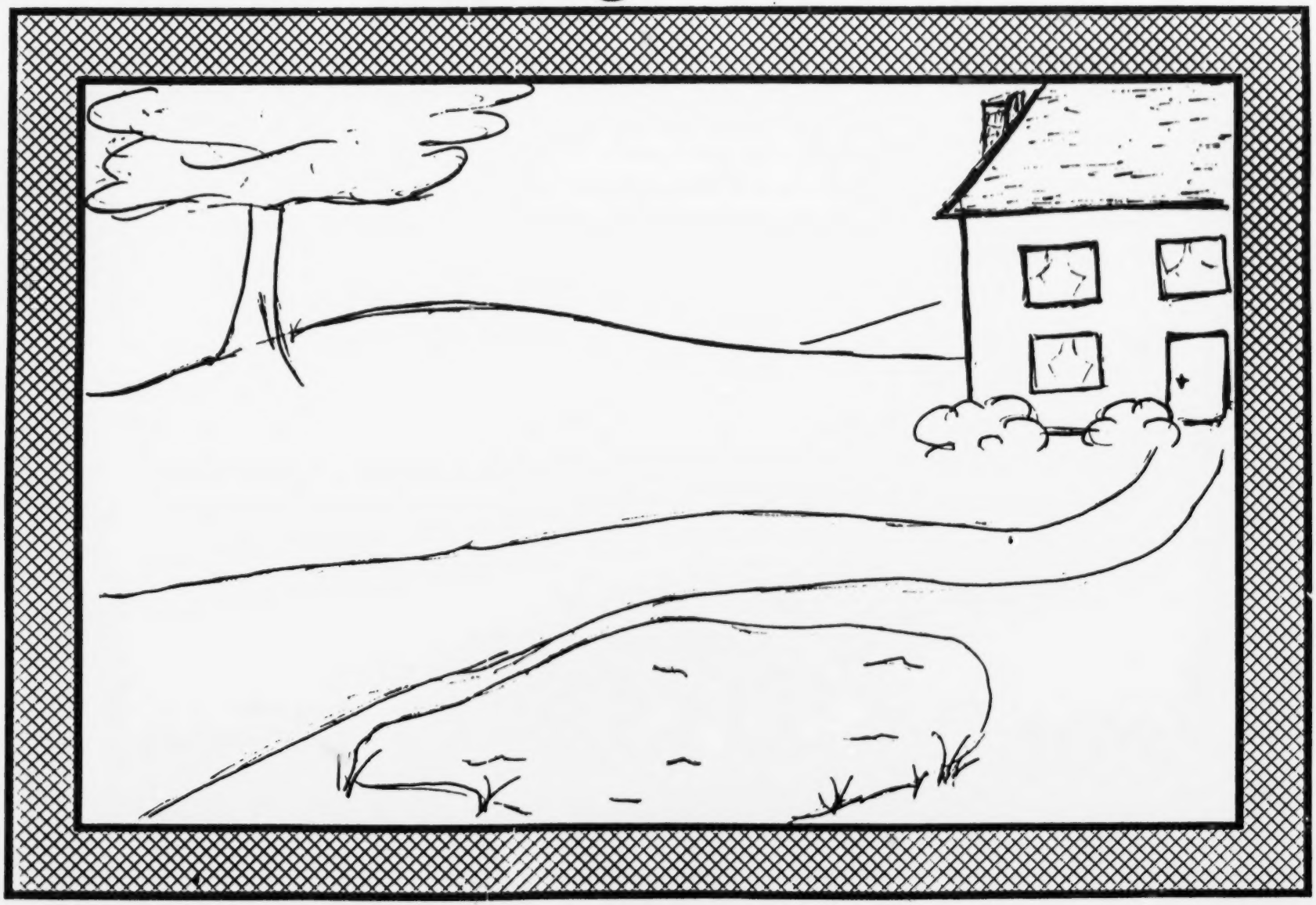
A. Using materials which can be snapped or bundled together (Unifix cubes, or popsicle sticks but not base ten materials) and place value mats, have student build a two-digit number. Ask the student to add or subtract a multiple of ten. For example, build 37 and add two tens. The teacher will record the problem and the solution to connect the student's demonstration with the manipulative.

B. Using Unifix cubes and a place value mat have the student build a two-digit number and add or subtract a multiple of ten as instructed by teacher. Student provides a written record of the problem and solution for a more advance level.

C. Given a problem with a two-digit number and a multiple of ten to add or subtract, the student can model the problem and record the resulting sum or difference. Have student explain what is taking place and how the recording relates to the model.

Story Mat

Visiting Friends



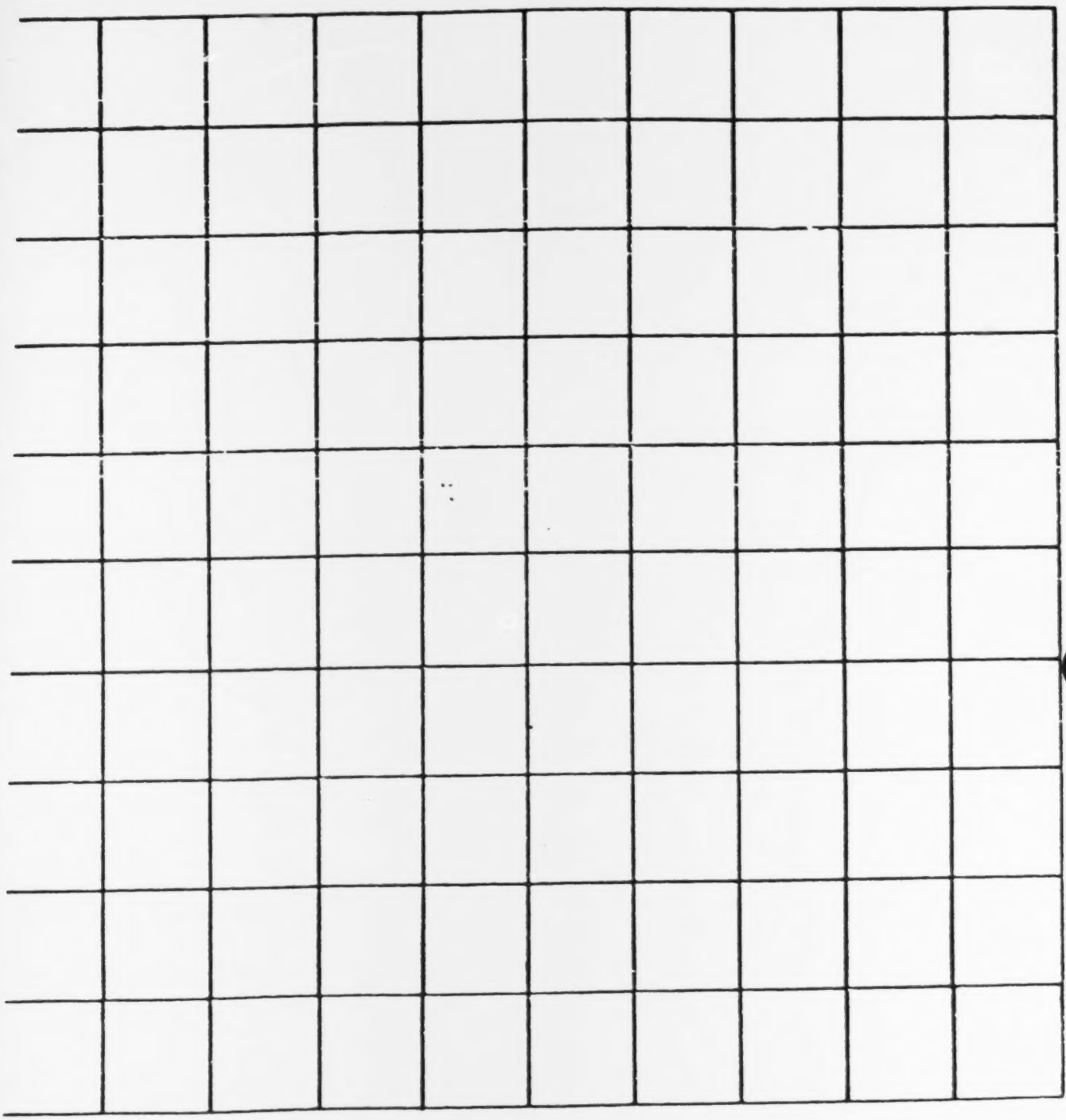
A-49

Place Value Mat

tens	ones
69	70

A–50

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



ADDITION TABLE

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Easier Facts

Medium Facts

Hard Facts

A-53

MATHEMATICS

GRADE 2

ASSESSMENT STRATEGIES

Completing Student Profiles

In recording observations about an individual student's knowledge of mathematical concepts and skills, teachers should not set aside specific "testing" time for the majority of the objectives. The purpose of ongoing assessment is to observe the student using skills in a variety of settings. Only for screening special cases or for noting objectives that are difficult to observe informally would individual or small group sessions be needed.

Initial observations should be recorded during the first semester for skills which have been taught. The suggested code for the profile is "N" for not yet, "S" for sometimes, and "M" for most of the time. Figure 1 explains the form.

Goal 1: *Broad goals are printed for each section of the profile*

1.4 Objectives to clarify the competency goals are printed in left-hand boxes	*Dates and codes*	*Comments reflecting observations during the first part of the school year ... included as appropriate*	*Dates and codes*	*Comments reflecting observations during the latter part of the year... clarification of performance*
1.4 Identify and use 10 more and 10 less				

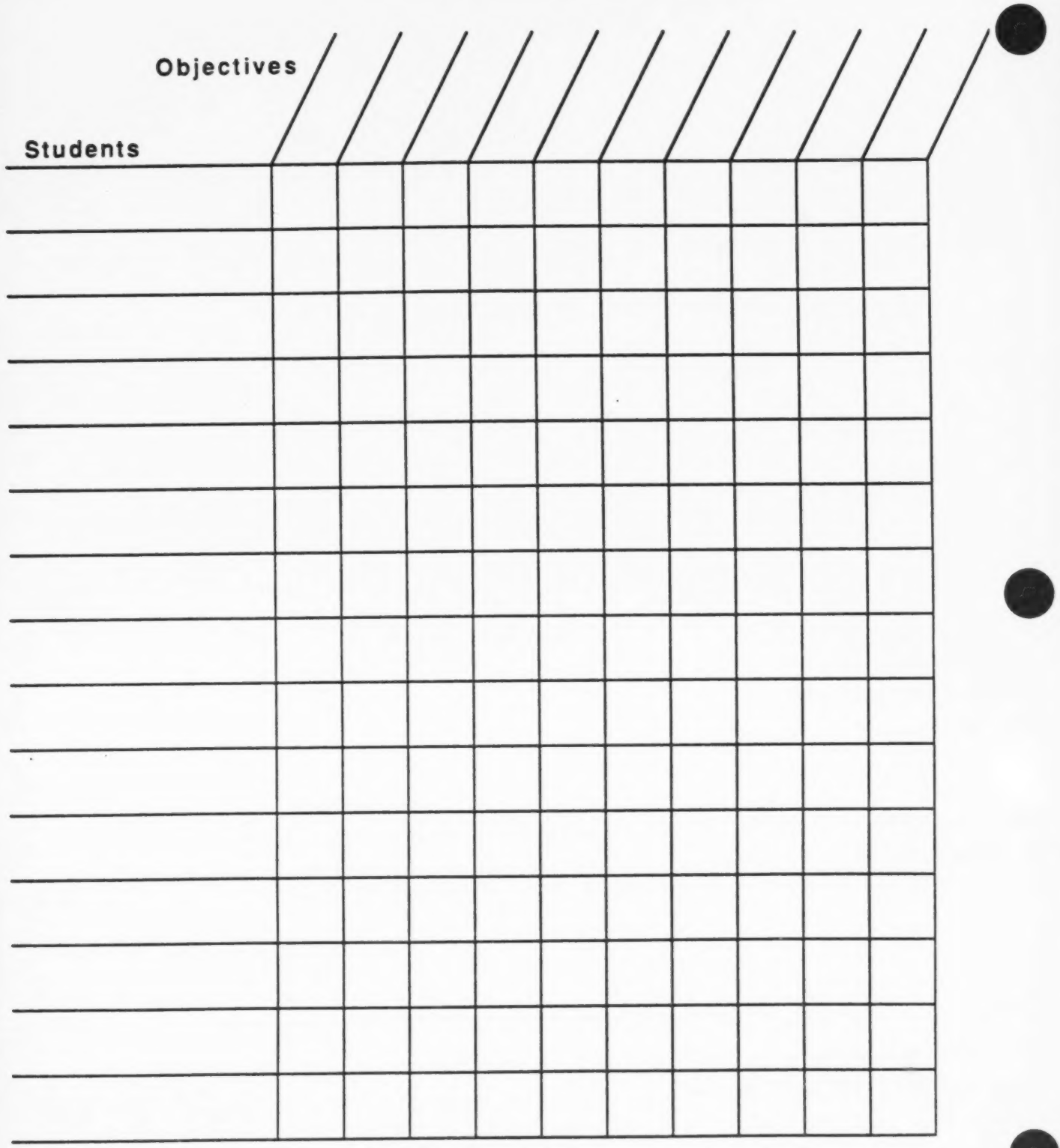
Figure 1

For each competency goal and objective, there are several suggested assessment strategies. While the suggestions are stated in terms of the teacher working with an individual to complete a task, the student most frequently will be working with other students. The teacher might ask a large group of students to complete a task and note specific students' performance. If lessons are presented to the total class and there are objectives that are not developmentally appropriate for some students at that point in the year, teachers should make appropriate notations. Similarly, if a student demonstrates more advanced understandings, an earlier and more detailed look at the child's performance will be helpful in instructional planning.

While primary teachers have always evaluated their students based on daily performance, recording these observations on a standardized checklist which relates directly to the Teacher Handbook will create a record of student understanding and progress that will be helpful in planning instruction as well as parent conferences. Samples of students' work will help illustrate notations on the profiles.

The final comments, recorded late in the school year (or whenever individuals exhibit consistent achievement of an objective), should attend to a student's independent efforts . Some objectives have appropriate oral or "hands-on" indicators, while others are appropriate pencil and paper tasks and will be assessed in written form. Chapter, unit, and summary tests from the state adopted mathematics textbooks may be used to supplement the teacher's observations. Many students will go beyond expectations in some goals and must never be denied opportunities for continued growth and challenge. While there will not be numerical scores for students, the profiles should show the extent of students' achievements in different aspects of mathematics.

Group Profile



Competency Goal 1: The learner will identify and use whole numbers to 100 and beyond.

1.1 Rote count beyond 100

A. Have child count, recording the upper limit. *It is not necessary to begin counting at one! The purpose is to note if child knows the counting sequence. Teachers may wish to begin by saying "How high can you count?" and allowing the child to decide where to begin.*

B. Ask student to write counting numbers in sequence when given the first three numbers.

1.2 Estimate; Use various counting strategies to determine how many

Counting strategies that students need to experience include 1 to 1 correspondence, tallying, counting on, and grouping. Teachers should model all of these throughtout the year. It is essential that students have a great deal of practice counting on from 10.

A. Ask child to count a set of 15-20 objects (concrete level). Given a worksheet (pictorial level), ask student to tell how many objects are pictured even when they are not grouped in tens and ones.

B. Given a known number of objects, ask student to count as others are added. "There are nine Unifix cubes in the bag. (Allow student to count to verify.) When we put these five cubes into the bag, how many will there be?"

C. Student applies counting strategies in computation. For example, in solving 8 + 3 the student may say "**eight**, 9, 10, 11".

D. Given a model of a two-digit number, the student will count on to add more to the set. At a pictorial level the student will not count the individual ones in a representation of ten but will count on.

1.3 Group tens and ones; read/ write standard numerals

All students should have opportunities to create as well as to use place value models. For most students it is during the second grade that the pattern of our place value system begins to take on meaning. Students are expected to use 0 to 9 to express numbers greater than the sets these numerals initially represented; they are expected to remember ones and tens (and hundreds) positions for digits and to recognize when the "5" represents five ones and when it means five groups of ten. Early place value models, loose objects that are grouped in cups or items to be bundled or snapped, are extended to include models that cannot be taken apart but must be traded. Making bean sticks is a popular activity which allows students to create their own base ten models. These bean sticks, made from popsicle sticks, pea beans, and white glue, can be assembled in a class factory. Groups of students can work in "shifts" on an assembly line to create and package individual sets for all children in the class. Models for numbers, however, should not be limited to what the teacher describes. Challenge students to find ways to tell about numbers and provide opportunities for them to explain their models.

A. Give student a group of between 25 and 50 objects. Ask child to tell how many by grouping them into 10's and 1's. Write the number as ___ tens and ___ ones as well as a standard numeral.

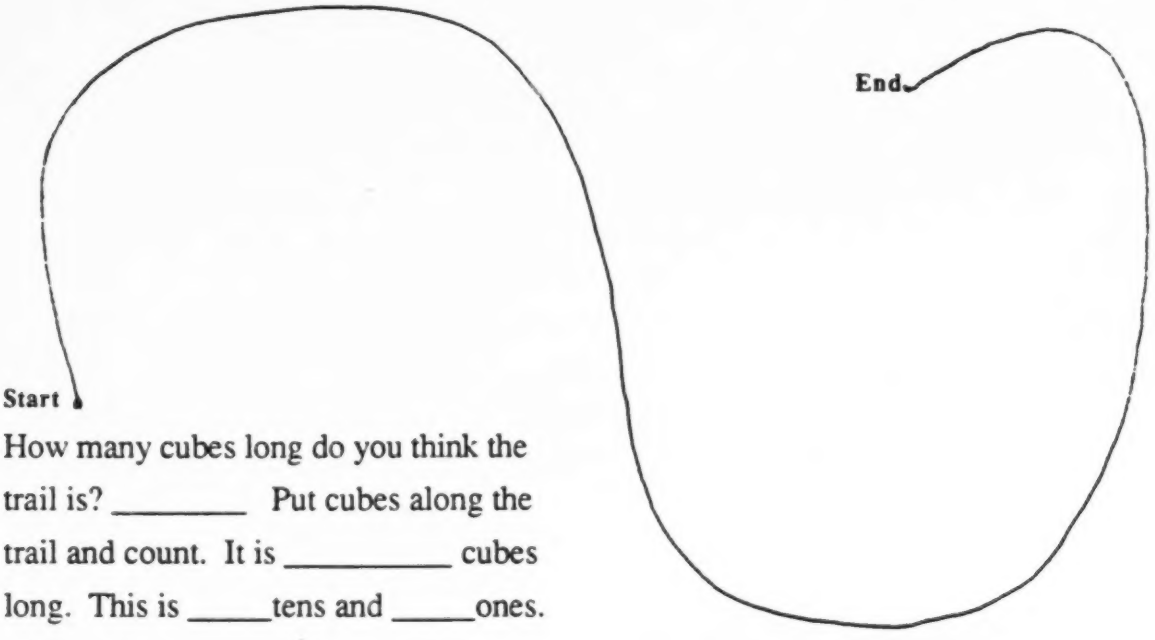
B. Give student a Unifix activity sheet (see samples on following page). Ask student to place cubes on the sheet, count, group in tens and ones, and tell how many. Write the number as tens and ones and in standard form.

C. Have the student model the same number with three different materials. Allow student to draw a picture as one of the representations.

D. Use edges of computer paper for base ten models. *Adults must precut the edges into strips of ten holes and small squares with a single hole to insure accuracy of the models.* Ask student to model four different two-digit numbers on a piece of dark construction paper that has been divided into four sections.

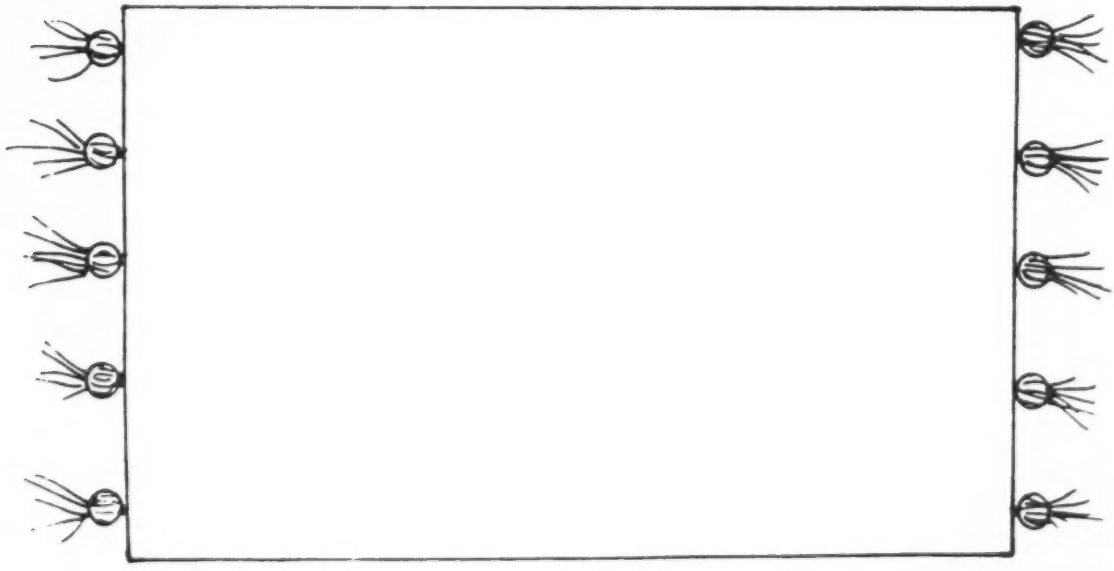
E. Have student build 26 using Unifix cubes or loose objects and cups. Give student eight more counters and ask the child to tell how many. Ask student to explain why two tens and six ones is the same value as one ten and 16 ones.

Unifix Activity Card 1: FOLLOWING THE TRAIL



How many cubes long do you think the trail is? _____ Put cubes along the trail and count. It is _____ cubes long. This is _____tens and _____ones.

Unifix Activity Card 2: FIX A UNIFIX RUG



How many cubes do you think it will take to completely fill the rug? _____

Make a design with Unifix cubes. Count them. There are _____ cubes. This is _____tens and _____ones.

F. Model a two-digit number on the overhead. Ask student to tell about the model and demonstrate how to write the number as tens and ones as well as in standard form. Show student a two-digit number and have child build the model.

1.4 Identify and use 10 more and 10 less

A. Have student build a number. Ask child to show one more and one less. Ask child to show one more and one less than a number marked on a hundred board. Ask student to tell what number is one more or one less than a given number. Have student write the number that is one more or one less than a given number.

B. Ask student to model 36 and then show ten more than 36. Have student show ten less than 36. Repeat with other numbers.

C. On a hundred board have student mark ten more or ten less than a multiple of ten (for example, 40). Using the hundred board, count by tens from numbers other than multiples of ten (for example, 28).

D. Ask student to write ten more or ten less than a given number. Repeat with other numbers.

E. Have student solve word problems involving ten more or ten less.

1.5 Compare and order numbers; identify missing numbers in a sequence

A. Have the student build two 2-digit numbers (or smaller). Ask child to explain which number is larger or which number is smaller.

B. Given two pictures representing numbers, ask the student to identify which is greater. Given three or more pictures representing numbers, ask the student to order them from least to greatest.

C. Given two numbers, ask student to circle the larger (or the smaller). Given a list, have student circle the greatest number (or the smallest).

D. Show student a sequence of cards with one missing. For example:

Ask child to tell what number is missing. Given a sequence of consecutive numbers, the child will fill in the blanks.

46, 47, 48, _____, _____, 51, 52

E. Given three numbers on cards (four or five for more advanced students), ask student to arrange the number cards from smallest to largest (or from largest to smallest). Use consecutive numbers until the student is consistent in being able to arrange them without difficulty. Give the student random numbers for a harder task.

F. Give student eight cards with sequential numbers (cards randomly arranged). Without looking, pull one card from the child's set. Ask child to put the cards in order and tell what card the teacher has. If the student's cards are in an unbroken sequence, ask what are the two possibilities for the teacher's card (the number before or the number after the student's sequence).

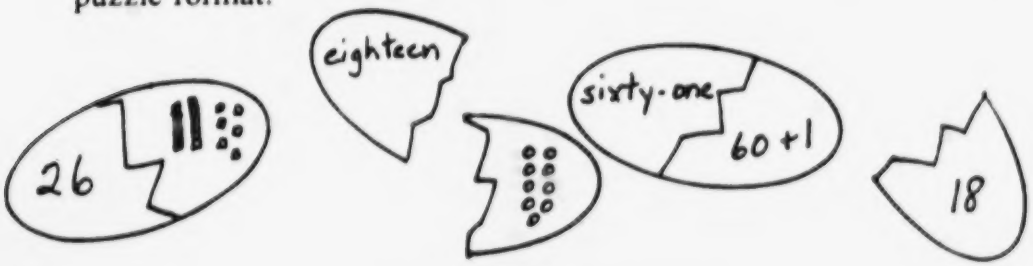
G. On the overhead cover five numbers on a hundred board. Ask student to tell what numbers are covered. On a hundred board have student cover five numbers in a pattern and ask student what additional numbers should also be covered? For example : 20, 22, 24, 26, 28 or 35,40, 45, 50,55.

H. After student can order cards correctly (sequential and nonsequential numbers), give numbers on a worksheet and ask student to write them from least to greatest or greatest to least.

1.6 Read/ write word names for numerals correctly

A. Have teams of students make collections of poem, song, and book titles which have number words in them. Ask student to read the titles that have been collected. *Students should be encouraged to help each other spell correctly and read the number words.*

B. Have student match cards with numbers to those with word names. Incorporate recognition of pictures of models of tens and ones in a puzzle format.



Put the dinosaur eggs back together.

C. Ask student to write word names for single digit numbers. Have student write word names for multiples of ten. Advanced students will write word names for any two-digit numbers. *Learning to write the number words when they are not being used in a context is a spelling lesson.*

1.7 Skip count by 2's, 5's,10's; group objects to count by 3's and 4's

A. Ask student to use objects to model skip counting by 2's, 5's, and 10's. Note if child groups loose objects into repeated sets. Ask student to (rote) count by 10's, by 5's, and by 2's.

B. Have student model patterns of counting by 10's, 5's, and 2's on a hundred board. Have student write skip counting sequences of 2's, 5's, or 10's. Given a worksheet with counting sequences, have student fill in missing numerals.

C. Observe if student uses tallying to count immovable objects or events in sets of five. "How many shelves in the room? How many doorknobs in the hall?" Count by 5's.

D. Have child make up to nine sets of three (or four). Have student determine how many by adding repeatedly.

E. On a hundred board have student mark every third (fourth) number, beginning the sequence with three (or four). *Skip counting by 3's (or 4's) does not mean to mark a number and then skip three (four) before marking the next.*

It means to mark the third number, beginning your counting with the number immediately following the one you have just marked. If six is marked, for example, the next marker is placed on the third number after six.

F. Ask student to color the pattern of counting by 3's or 4's on a hundred chart after the student has identified the pattern with markers. *Counting sequences colored on hundred charts and displayed around the classroom help students focus on the patterns in the number system.*

G. Ask student to count (rote) by 3's or 4's. Have student write the counting sequence of 3's or 4's. *It should not be a second grade expectation that children will memorize these sequences. Rather the focus should be on seeing the patterns and understanding the idea of skip counting as an easy way to add similar sets. This is a readiness - for - multiplication idea that needs to be well-developed in second grade at a concrete level.*

1.8 Name nearest multiple of 10 on a number line when given a number

A. On a "walk-on" number line ask student to stand on six. "How many steps (units) would it take you to get to ten? How many steps (units) would you need to take to get to zero? When you stand on six, are you closer to ten or to zero?" Repeat activity using other numbers.

B. Give student a teddy bear counter (or some other marker) and a desk top number line from zero to twenty. Have student place marker on a designated number. Ask if the marker is closer to zero, to ten, or to twenty. Ask student to explain what the possible responses are if the marker is on five or fifteen.

B. On a desktop number line marked from 18 to 53 have child place a marker on 28. "What is the nearest multiple of ten before 28? What is the next multiple of ten after 28? If your marker is on 28, to which ten are you closer?" Repeat with other numbers, always using a marker for as visual an experience as possible.

1.9 Identify odd and even numbers using objects

Children need to construct new vocabulary in which "even" means that each has a partner and "odd" means that there will be one without a partner. Focus first on numbers to ten.

A. Have students line up with partners. Does each child have a partner? Is the number of students present today odd or even?

B. Have student take a handful of objects (Unifix cubes or the larger pattern blocks are a good size). Ask student to show if there is an odd or an even number of counters. How many blocks did you pick up?

"Seven is an odd number because there are three pairs and one left over."

C. Show the student a numeral (vary to make appropriate for the student). Ask the student to predict if the number will be odd or even. Why do you think this? Ask the student how to verify the prediction.

D. Ask student to make a picture to show if the number nine (or another appropriate number) is odd or even. *Observe if the student has a strategy for verifying odd and even when manipulatives are not available.*

E. Ask if the student can suggest some rules about which numbers are odd and which ones are even. *Some students may not have reached a level of maturity to be able to do this. It is not necessary at grade two to memorize examples of odd and even numbers. The process of defining and determining if a number is odd or even should be the focus.*

1.10 Divide regions/ sets into halves

Early experiences with recognizing (and dividing) regions and sets that are in equal and unequal parts provide a foundation for naming fractions as "equal" parts of a whole. Through hands-on activities children will discover that the size of each part decreases as the number of equal divisions increases. For example, a slice of a large pizza that is cut into two equal pieces is larger than a slice of the same sized pizza that is cut into six equal pieces. It is important that students focus on"the whole" as they begin talking about fractional parts. While the primary focus is on halves, other fractions will appear in the context of the classroom. Set models may be easier for some students than models of geometric shapes (regions). Likewise, students may recognize one model (region) but not others (sets or

parts of a line). It may be easier for students to recognize pictures of fractional parts than to the same divisions by themselves Similarly, recognition of regional models may prove easier than creating the regional models. Students should always be encouraged to explain their thinking.

A. Give the student a workmat showing two animals and an even number of kernels of corn (counters) . Ask student to place one-half of the corn in each animal's bowl.
B. Given a cracker, ask student to cover one-half of it with peanut butter. Have child cut a sandwich so that two students can eat one-half.
C. Using the 2-color counters, ask student to place 12 (and other appropriate numbers) red ones on the desk. Turn one-half over.
D. Given a yellow hexagon and a blue parallelogram from the pattern block set, have student show the blocks which are one-half of each of these. Ask the student if there is a block to cover one-half of the red trapezoid *(no).*
E. Have students fold different size pieces of paper into halves.. Ask student to color one half. Are all of the one-half colored pieces the same size?*(no)* Ask student to explain why they are not.

1.11 Group objects into ones, tens, and hundreds

Place value concepts develop slowly as students learn that ten 10's make a hundred. Understanding "hundredness" as a collection of ten 10's extends the pattern that one 10 is a collection of ten 1's. When children begin to understand this pattern at a concrete level, they have a reason for recording the numbers in the standard format adults call place value. All students should be exposed to numbers beyond 99 at concrete levels. However, students need a thorough understanding of tens and ones at concrete, pictorial, and abstract levels before they are expected to work with three place numbers independently.

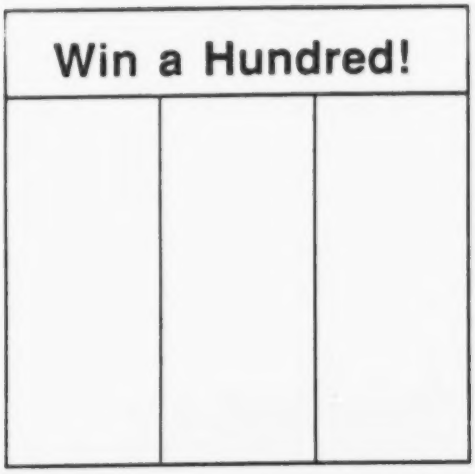
A. Use a blank 10 x 10 grid. Have students place ten Unifix cubes of the same color in each row. "How many cubes did you use to cover the

board?" (Alternate colors by rows.) Snap the cubes in each row together. "How many are in each group? How many groups do you have? How many tens are there in a hundred?"

B. Have student model 100 as ten 10's in a variety of ways. Display ten paper chains of ten loops each, ten necklaces of ten beads, ten cards with ten pennies taped (stamped) on each, ten cards with ten seeds glued on each, ten chains with ten paper clips, ten trees with ten leaves on each, etc.

C. Give students a bag of small objects. Ask children to group these into ones, tens, and hundreds, using cups for tens and box tops to hold ten 10's for the hundreds. Ask children to explain what they have done. *Because of the number of items, students should work in pairs, verifying each other's counting. Ask the students to explain the results. The teacher should show the students that one group of a hundred, four groups of tens, and two ones may be written as 142. The teacher also needs to discuss that another way to name the number would be 14 tens and 2 ones. Ask why is this true.*

D. Using a three-part place value mat have students play games such as "Win a Hundred." Students can use bean sticks and loose beans for tens and ones. A hundred is a raft (flat) made of ten bean sticks. Commercial base ten materials, bundles of toothpicks, chains of paper clips, etc. work equally well.



Each student has a game board and at a turn rolls a die which has been numbered 4, 5, 6, 7, 8, 9. The student collects the number of beans indicated on the die. *Three sets of number cards shuffled and reshuffled as needed could be used instead of a die.* Student trades ones for a ten (a bean stick) whenever possible. The winner is the first student in the group to trade ten bean sticks (10 tens) for a raft (flat).

1.12 Model 3-digit numbers; choose/ read/ write correct numerals

A. Using a place value mat, have student build a number. For example, have student put 2 hundreds, 4 tens, and 7 ones on the mat. Ask student to name/ write the number.

B. Play "Out Number Your Neighbor". Each student draws 3 digit cards and models the numbers on the hundred board.

Version 1: The first card tells how many ones; the second card tells the numbers of tens, and the third card tells how many hundreds. The students model the numbers on the place value mats. The player with the highest number (or lowest number) wins a point. Play continues until one student has 10 points.

Version 2: Students may arrange their cards to build either the highest number or the lowest number they can. Children will designate which digit identifies the value of each place. The winner of both the high number and the low number wins a point when the models are compared.

C. Given pictures which model three digit numbers, tell and write the number in standard form. *Note if the student recognizes and utilizes the groupings that are displayed or if the student tends to count individual parts.*

Competency Goal 2: The learner will demonstrate an understanding and use of geometry.

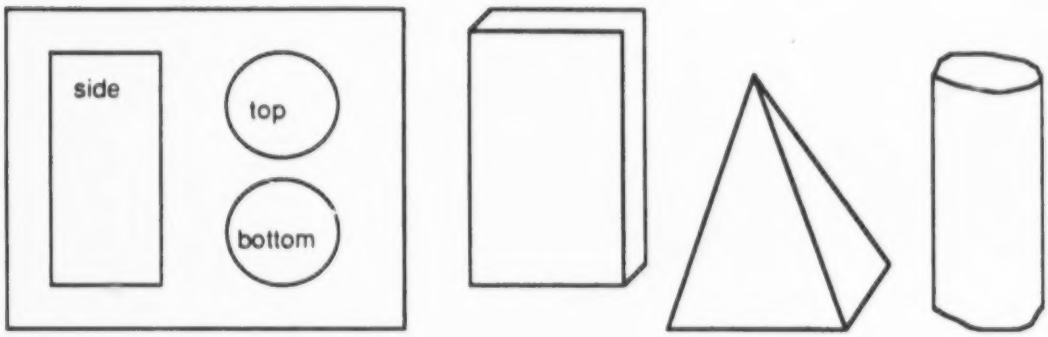
2.1 Identify/ make/ plane and solid figures, including cylinders, using appropriate vocabulary

Most students will recognize circles, squares, triangles, rectangles, diamonds, boxes (cubes), balls (spheres), and cones. New vocabulary would include trapezoid, hexagon, oval, parallelogram, cylinder, and line segment. Lessons should focus on the characteristics (attributes) of the figures. The development of a working vocabulary and opportunities to create models of figures are important goals for grade two.

A. Have student sort then describe blocks (ex. Relationshapes, attribute, geoblocks, and real-life models). Explain the grouping.
B. Have student make figures on geoboard, telling about each figure. How many different hexagons can you make? Make a square and a trapezoid on the geoboard; how are they alike? How are they different? Make three different shaped triangles.
C. Using straws, tinker toys, toothpicks, clay or other materials, have student build models of plane and solid figures. Ask student to organize the models in some way and make a display.
D. Given models, have student identify the one with the most (fewest) corners or edges. Ask student to explain what a corner or an edge is.
E. Place one figure in the student's hands (child has eyes closed). Have student identify the figure by touch. Place several models in a box or bag. Ask student to find, by touch not by looking, a certain figure.
F. Have students make books about figures including pictures from magazines as well as their own drawings. Direct students to write (tell assistant) a sentence about each page.
G. Have student solve "Who am I?" puzzles. For example: "I am a solid figure. I have six faces. All of my faces are the same (congruent). Who am I?"

H. Have child hide a figure behind a folder. The task is to describe the figure to others, not naming the shape, so that other children can identify the figure or (at a more advanced level) draw a picture of the model.

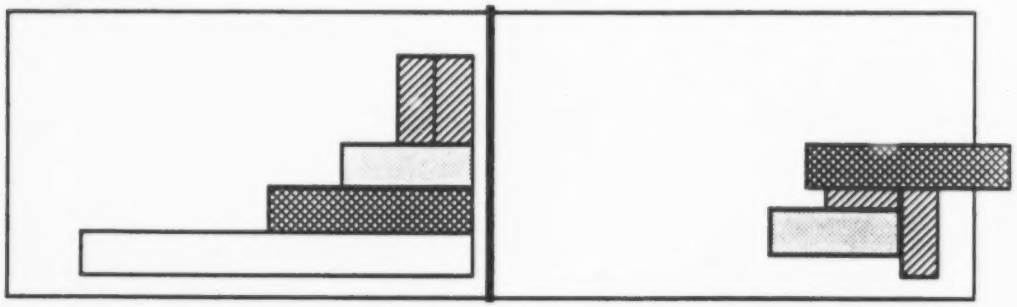
I. Ask student to identify which solid matches the "blueprint" the teacher has drawn.



2.2 Identify and make figures with line symmetry

A. Have student fold paper and cut out figure (pumpkins, bugs, hearts). Draw line of symmetry on the fold.

B. On a work mat that has a line drawn to indicate the line of symmetry, ask student to build with pattern blocks or Cuisenaire rods a symmetrical design. Work with a partner. The second student copies on the right side of the line of symmetry pieces that the first student has placed on the left of the line.



Student makes design. Partner uses same blocks to build the reflection of the left side.

C. Give student pictures to cut out. Ask child to decide if the figure is symmetrical and to locate lines of symmetry by folding the figure.

D. Give student paper which has alphabet letters printed as capitals. Ask student to find letters with lines of symmetry. *Children will discover both horizontal and vertical lines of symmetry.* Tell how you know the letters are symmetrical. Do some letters have more than one line of symmetry? *An extension: challenge student to write a word in which all letters are symmetrical.*

2.3 Match congruent figures

A. Use cookie cutters to make congruent shapes out of play dough, clay, or real dough. Ask student to explain what congruent means.

B. Make several designs with pattern blocks or other materials. Ask child to identify which ones are exactly alike (congruent). Ask child to make three more just like the one the child identifies as the favorite.

D. Play "Copy Cat". Seat two students side by side. One child makes a design and the other copies it. Repeat and take turns. Students should talk about their designs.

E. Cut several different shapes from construction paper. Ask student to match those that are exactly alike. What does congruent mean?

2.4 Replicate 3-dimensional designs using models

A. Make a 3-dimensional design with blocks. Ask student to copy the design. Make a 3-dimensional design with Unifix cubes. Ask student to estimate how many cubes are needed to copy the design (How many are hidden?) and then reproduce the design.

B. Have one student make a design that is hidden from a partner. The first student describes the design and the second student tries to build the design from the description.

C. Show student a picture of a 3-dimensional design (blocks, for example). Ask student to make the design.

2.5 Recognize square corners/ geometric figures in the environment

A. Have child identify specific shapes in the classroom. For example, where do you see a circle? Where do you see a cube?

B. Have student group shapes by the number of sides or edges or faces. Have student find all shapes which have right angles. *Students should recognize right angles but do not need to memorize that they are 90 degrees. Since a degree is a measure of how much something turns from a given position, students should have opportunities to "act out" the opening and closing of the rays which make angles. They need to explore geometric ideas using Logo and will enjoy taking the role of the turtle to predict its movements before going to the computer. Many students will call the right angles square corners.*

C. Ask student to identify three right angles somewhere in the classroom. Have student model a right angle on a geoboard, with tinker toys, or with other materials. Ask child to make on a geoboard a figure that has a right angle.

D. Show student a picture from a magazine. Have child to outline basic shapes.

E. Ask child to identify basic shapes in common objects. For example, "What shape are most students' desk tops? The tops on gallon milk jugs are what shape? What shape are the highway signs that say 'Yield'? What shape are most dice?"

Competency Goal 3: The learner will demonstrate an understanding of classification, pattern, and seriation.

3.1 Compare similarities and differences

A. Ask two students to stand before the group. Have a child tell ways the students are alike and ways they are different.

B. Place objects (toys, math manipulatives, etc.) in a box. Have student choose any two (or three). Ask student to tell (write) ways they are alike (similar) and ways they are different. Compare two animals or two buildings.

C. Make a collection of leaves. Organize them in some way and tell how they are alike and how they are different. Compare rocks in a collection .

D. Use models of plane and solid figures. Have student choose two figures. Ask student to describe similarities and differences using as many "math words" as possible.

3.2 Classify by more than one attribute; describe rules used in sorting

When students group objects, pictures, or symbols, they will continue to organize many items by one rule. For example, putting the silverware away has the child sorting forks, spoons, and knives, etc. Grouping foods by meats and fish, dairy products, fruits and vegetables, and grains could be considered as grouping by one rule, food groups. If students sub-divide these groups, they would be applying a second rule. Describing attributes is an important first step in being able to sort (classify), and the language focus should be continued as students group objects.

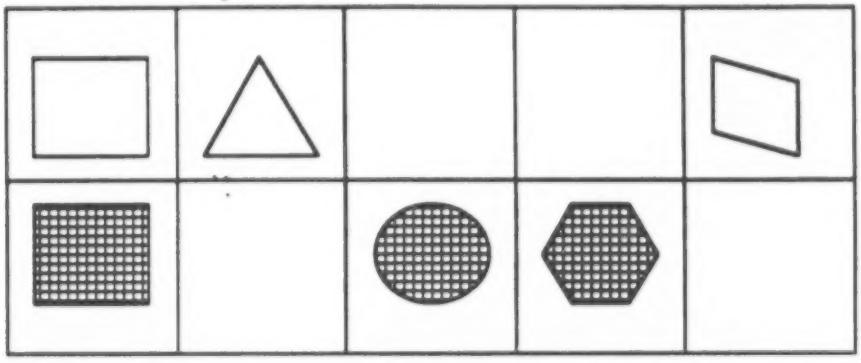
A. Ask student to line up according to 2 or more attributes. For example, "Boys who have on jeans and tennis shoes get in line. Girls who have on red and have belts may get in line."

B. Make an attribute train using Relationshapes or other attribute materials. Ask child to place a shape in the train that is different in two and only two ways from the previous shape. Explain the differences.

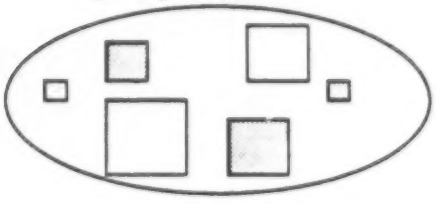
C. Ask student to sort collections/ ideas that relate to other content areas: leaves, animals, vehicles, types of homes, acceptable actions in settling differences, books being returned to the library, lunch tickets by girls/ boys and menu choices, etc. How did you organize the materials?

D. Show student materials which have been grouped. Ask child to describe why they are arranged in this manner. Ask to which group a new object belongs.

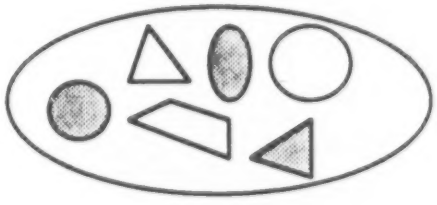
E. Ask student to place the appropriate blocks on a simple matrix. Have student create other puzzles.



F. Use sorting rings of yarn (Venn Diagrams) to show groups and intersections. Give students attribute materials and 2 rings. Suggest a way to group.

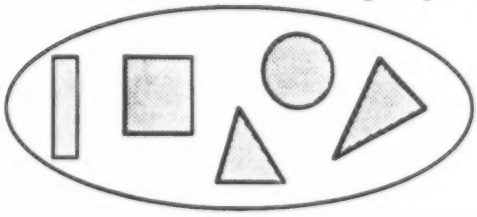


Squares

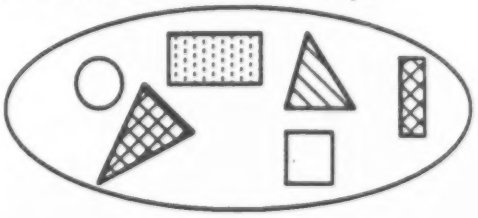


Other Shapes

Then ask the child to group the same materials in a different way.



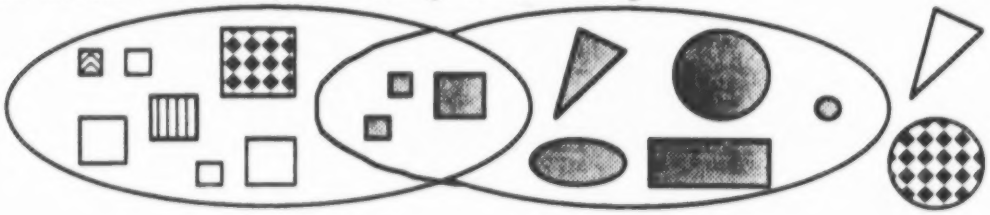
Blue



Other Colors

Combine the student's sorting idea and the one suggested by the teacher.

Ask the student what would be placed in the overlap of the circles ? *Blue squares would go in the intersection.* Where would red circles go?*Shapes that are not blue or are not squares would go outside the rings.*



Squares Blue Pieces

G. Use attribute activities such as those found in the book, The Fun World of Relationshapes PATCHES. Different sorting formats, such as tree diagrams, Venn diagrams, and matrices, are modeled. The book comes with the Relationshapes in the North Carolina Manipulatives Kit (Primary Level).

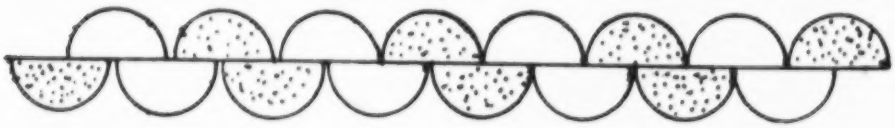
3.3 Define and continue patterns; translate into different forms

A. Show student a pattern made with objects. Ask the student to identify the pattern unit (that which is repeated). What will the next four pieces be? *Have student complete the same task with patterns pictured on a worksheet after the students have concrete experiences with many different materials. Beware of allowing children to string numerous different blocks together and never focusing on the basic ideas of pattern as they create their own patterns. They will enjoy working with the manipulatives, but they may not be developing an understanding of pattern unless they are exposed to discussions about the concept. Students should be able to tell about the patterns they create and define the term "pattern" by giving different examples.* Show student a pattern such as AABBAABBAABB. Ask child to model the same pattern with actions, with blocks, and with words.

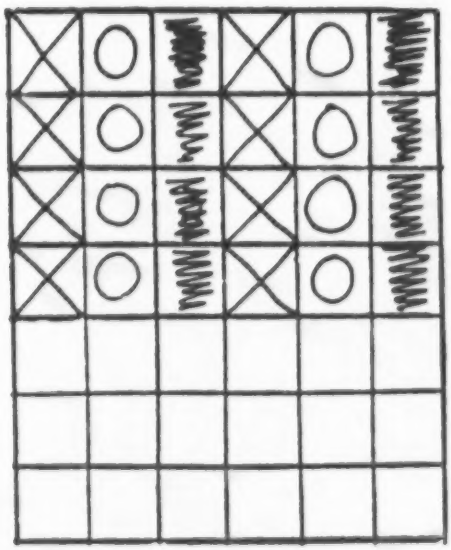
B. Have student create patterns and make bulletin board borders (use adding machine tape, construction paper, etc.). Use templates, stamps, and cut-out shapes. Ask student to explain what his pattern unit is and to tell about

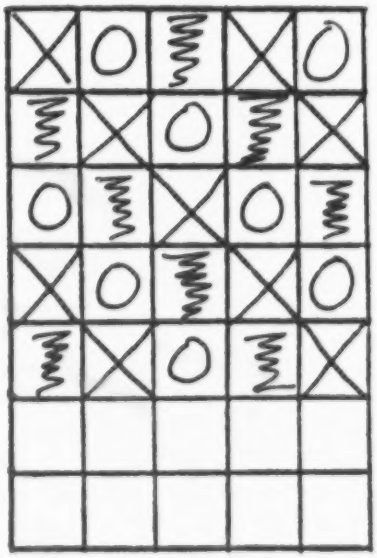
its repetition. Students can create pattern endpapers for the books they make or pattern "frames" to display their work.

C. Have student cut out 20 circles of the same size in one or two colors. Cut each circle in half. "What patterns could you create with these semi-circles?"



D. Have the student make wrapping paper for a special occasion. For a student who has difficulty creating a pattern unit tell the child the pattern to use, but allow each student to interpret it in a special way. *Notice that if the number of squares on a row of the paper is evenly divisible by the number of elements in the pattern unit, a striped wrapping paper is created. If the number of squares in a row is not evenly divisible by the number of parts in the pattern unit, a diagonal design appears.*





E. Have students discover skip counting patterns on a hundred board. Use Unifix cubes or other markers to cover the numbers. Have student color the same pattern on a 100 chart. Ask student to write number sequences that were colored.

F. Place 15 counters (skip counting by 3's or 4's) on a hundred board on the overhead. Have student place counters on a board at the same time. Ask child to place the next ten counters on the board and explain the reasons for their placement.

G. Have students create unusual patterns on their hundred boards as enrichment activities. For example, "Begin at three; cover five; cover eight; cover ten; cover 13; cover 15. What is the pattern?" or " What rule did I use to place the markers?" *(plus two, plus three)* Have students examine each other's patterns and try to tell the next five numbers. Be certain that students can explain the patterns they create.

3.4 Identify classification and patterning in the environment

A. Ask student to point out patterns in the classroom or find pictures of patterns in magazines (clothing, wallpaper, floor and ceiling tiles, bricks, sidewalks).

B. Ask student to name ways in which things have been sorted. Give child some examples, if necessary, to clarify what you are asking for (clothes in certain drawers, vegetables in the grocery store, books in library).

C. Ask child to describe how toys might be placed in the store or at home so that they are organized in some way. Have student organize something in the classroom (materials at the art table, the puzzle center, etc.).

D. What might these numbers be: 786-3278 or 843-8524 or 923-7391 ? *(telephone numbers)* What might these numbers be: 5/2/88 or 11/9/88 or 9/16/88 ? *(dates)*

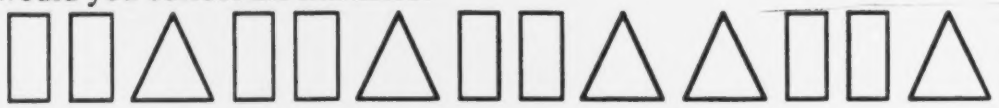
E. Have student identify and record examples of patterns through crayon rubbings. *(Check the bottoms of tennis shoes!)*

F. Have student find examples of patterns in books, songs, and poems. Ask student to find examples of patterns in other content areas (calendar, seasons,etc.).

3.5 Identify/ correct errors in patterns

A. Show student a picture (diagram) of three places set at a table. Ask student to tell what is not correct (for example, silverware not properly placed). "How could the mistake be corrected?"

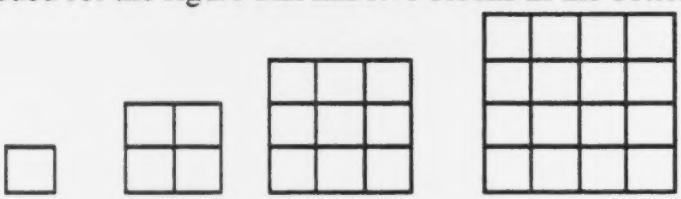
B. Given a pattern in blocks, have student correct the errors. Ask student to identify mistakes in patterns that are pictured on workbook pages. "How would you correct the mistakes?"



C. Have students discuss in a group the patterns in place value.

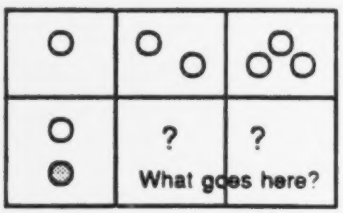
3.6 Use patterns to continue numerical sequences

A. Use blocks to show a growing pattern. Ask child how many blocks will be needed for the figure that has five blocks in the bottom row.



B. Combine spatial problem solving with growing patterns.

C. On a hundred board cover 1, 3, 5, 7, 9, 11. Ask student to cover the next three numbers in sequence. Skip count by 5's starting with the number three.



D. On a hundred board cover a pattern such as 2, 7, 12, 17, 22. What are the next three numbers which need to be covered in the sequence?

E. Write (or have student cover on a hundred board if needed) 94, 84, 74, 64, 54. Ask student to write (tell or cover) the next three numbers.

3.7 Order objects and events; use ordinal numbers

A. Ask student to tell the steps (first, second, third, etc.) in making a peanut butter and jelly sandwich or to retell a familiar story. "What happens first? Second?

B. Given containers, ask student to order them according to how much they will hold. Allow student to verify the arrangement using water, sand, rice, beans, etc. Have student explain how the task was accomplished.

C. Given a set of four or five pictures, ask child to sequence them correctly.

D. Have child build a tower with cubes as the teacher gives the directions. "First floor is blue. The second and third floors are red. The fourth floor is yellow. The fifth floor is brown. The sixth floor is green. The seventh is black and the eighth is orange. If you go to the brown floor, on which floor are you? What color is the third floor? The floor below the seventh is what color?" etc. If the student is not confident in answering the questions, have the child turn the tower on its side to become a train with the first car being blue. Ask the student questions about the cars in the train. "The fourth car is what color? What place in line is the green car?" etc.

E. Give student three objects and a balance. Ask child to order them from lightest to heaviest. Ask student to explain how this ordering might be carried out since the measuring device is a balance. "Could you figure out the lighest to the heaviest with five things?"

F. Given sentence strips with statements about completing a task (for example, how to bake a cake or how to wrap a gift), ask student to put them in the correct (logical) order.

G. Ask child to order coins according to their values. Ask child to explain how this order would differ if coins are being arranged by size.

H. Give student "purses" with different coins to order according to the amounts of money they contain.

I. Play Ordinal Concentration. Shuffle 16 cards (eight matching pairs) and place them in a row. Two or three students take turns asking to see different cards, describing them by their ordinal position. For example, "Show me the fourth and the twelfth cards." When a match is made, those cards are removed, changing the ordinal positions of the remaining cards. The winner of the game is the student with the most pairs. A variation of the activity incorporates locating coordinate positions when the cards are placed in a 4 x 4 matrix. Then the child must ask to see, for example, the second card in the third row.

J. Give student objects and directions to place them in specified places. "Put the book on the fourth shelf. Place the crayons on every fifth desk."

K. Direct students to place themselves in line at specified points. "Julie, you need to be seventh in line; and Sean, you need to be fifteenth."

Competency Goal 4: The learner will understand and use standard units of metric and customary measurement.

4.1 Estimate one inch or centimeter; measure length to nearest inch or centimeter

Since students learn to measure with standard units during the second grade, they need to have a "ballpark" idea of the units they are using. Learning the length of an inch or a centimeter should include activities which relate the units to the child's body. Students should not convert units from one measurement system to the other.

A. Give student a piece of tag-board one inch long. Ask student to name three things in the classroom that are about an inch long. Compare to see if estimates are reasonable. Repeat the activity with a centimeter long object (a centimeter cube is easy for student to hold).

B. Make collection of one-inch items. Ask student to list three objects at home that are about one inch (centimeter) long. Have student check and report back to the teacher or the class.

C. Show the student a collection of objects. Ask the student to identify which objects are about one inch long by sorting objects into groups: less than one inch, about one inch, and longer than one inch (for example, a paper clip, a short pencil, or crayon, Dentine gum, a hair clip, a pair of small scissors, a button, etc). Compare with a one-inch model to verify the sorting.

D. Given a paper with different length lines drawn on it, ask student to tell which lines are about one inch long.

As standard units are introduced, students need to have sufficient experiences to understand why the need for standard units exists. They need to have a feel for the units themselves; they need to know how and when to use different forms of measurement; and they need to be able to choose the appropriate measurement devices. Activities with inches will lead to an introduction of feet and yards during the second grade just as centimeters should introduce decimeters and meters. Sudents should make their own measuring tapes and inch rulers as a measurement lesson. Use one-inch segments in two colors and have students tape the pieces together in an AB pattern to make a flexible measuring device 12 to 18 inches long. Taping pieces to a ribbon makes the measuring device sturdier. Ask students how

they might number the tape to make measuring easier (number each segment). At an appropriate time repeat the activity making a rigid ruler by gluing or taping one -inch strips to a piece of tag-board. Talk with students about how to number this ruler. What happens if the first one -inch piece is not glued at the end of the background?

E. Ask student to estimate the length of three objects and then measure the objects to the nearest inch/ centimeter. *(Do not ask student to convert inches to centimeters or vice versa.)*

F. Using the homemade measuring tape have students measure each other. Ask student to compare the different measurements: arm with a leg; foot with a hand; circumference of head with arm, etc.

G. Given a workbook page with objects and rulers pictured, tell the length of the objects to the nearest inch/ centimeter. Given a workbook page with pictures of objects, measure them to the nearest inch/ centimeter.

4.2 Measure capacity to nearest cup

A. Give student a large container with macaroni (water, sand, rice, or beans). Ask student to determine how many cups there are.

B. Ask student to estimate and then measure the number of cups different containers will hold. How many cups will a quart container hold? *(explore, not memorize!)*

C. Ask student to determine if a container is more or less than a cup. *Compare containers as a means of developing a sense of their capacities. Do not expect student to memorize ounces in a cup.* Give examples of things which come in containers that are about a cup.

D. Ask student to measure a given number of cups of macaroni (or other materials) into a pot.

F. How many small juice cups can we fill with a quart of lemonade? *(Show student the quart.)* Will we be able to fill more or fewer measuring cups than juice cups with a quart of liquid? Predict and then test your answer.

4.3 Estimate number of smaller units contained in larger

A. Give student a pattern block outline. Ask child to estimate how many blocks will fill it. Have student complete the puzzle and count. Ask child if there is any way to complete the same puzzle using more blocks. Would there be a way to use fewer?

B. Give student a template or a pattern. Ask child to estimate how many of the figures could be cut from a piece of construction paper. Have student explain and tell how to check the prediction.

C. Ask student to estimate how many "bracelets" of a certain length *(show student length)* could be cut from a given string.

D. Ask student if more cubes of one size would fit into a box than cubes of another size. *(Show the two sizes of cubes.)* How many Unifix cubes could we place in a shoebox? How could you see if your estimate was reasonable?

4.4 Weigh objects to nearest pound or kilogram

A. Use a balance and a one-kilogram weight (or a one pound weight). Ask child to determine if objects are heavier or lighter than one kilogram (one pound). Record the findings on a chart.

B. Use a bathroom scale. Have student weigh. Ask student to tell the weight. Record the weight and compare in three months.

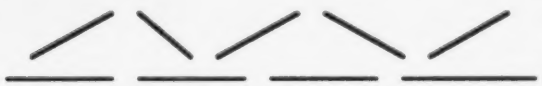
C. Bring items from the grocery store. Have child weigh the objects. Compare the results with the printed weight on the packages. Why might the numbers not be exactly the same? Have student order four items according to their mass.

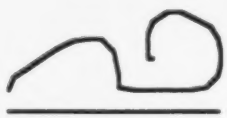
D. Give student four objects. Ask child to estimate which is heaviest and which is lightest. Have student weigh objects to check estimate.

4.5 Explore conservation of length, capacity, and weight

Mastery is not the primary goal of this objective; the purpose is exploration. (See the discussion of conservation in the grade one strategies.) Brief examples help to clarify conservation-type tasks: the length of a string is not affected by arrranging it in a different manner; the amount of water does not increase (or decrease) when poured from a tall, thin glass into a short, fat one; the weight of a ball of clay does not change if the clay is flattened.

A. Make a zigzag path with toothpicks or paperclips. Ask student to make another path, straight not crooked, that would be the same length. Ask student to explain about the path being created. *(Students who do not conserve length will not focus on the number of toothpicks or paper clips in the first path but will match the length in a straight line.)*





B. Show children a crooked path that a little bug might travel. *(yarn glued to construction paper)* Ask student to cur another peice of yarn that would make a path the same length for the bug's friend to travel.

C. Provide different containers for students to use for experimenting with capacity. Ask them to predict where the beans (rice, water, etc.) will come to in different containers. Ask children to explain why they make certain predictions. Discuss the results of the experiments.

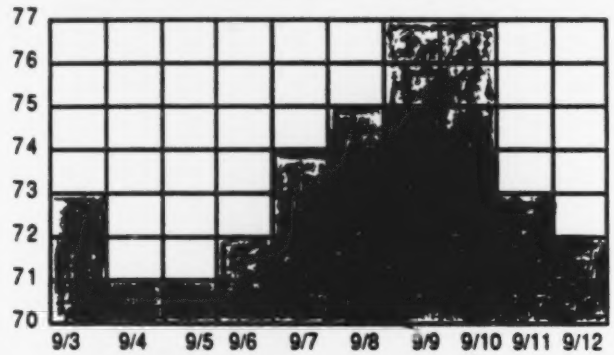
D. Have students count the number of scoops it takes to fill a peanut butter jar with lima beans. Will it take more, fewer, or the same number of scoops of unpopped popcorn? Explain your reasoning before and after experimenting. *(Will there be the same number of objects, beans and popcorn?)*

E. Attach Unifix cubes in sets of five to use on a balance as nonstandard weights. If it takes two five-sticks to balance an object, how many loose Unifix cubes will it take to balance the same object? How many loose Unifix cubes will it take to balance three five-sticks?

4.6 Read Celsius and Fahrenheit thermometers

A. Using a demonstration thermometer have the children act out the way they would feel at given temperatures. For example, at 0^0C (30^0F) the children would shiver, but at 25^0C (80^0F) they would pretend to be hot. Reverse the activity and have some children pantomime being hot, cold, cool, warm, etc. Another student would make the thermometer read correctly.

B. Have real thermometers inside and outside the classroom. Ask child to read the temperatures at the same time each day. Record the temperatures on a chart or graph. *This information can be used for a graphing activity in which students collect data and keep records over a long period of time.*



Fahrenheit temperatures at 9:30 a.m. during the first two weeks in September

C. Give student a drawing of thermometers. Have child color given temperatures. Show pictures of thermometers. Ask child to read temperatures. Match scenes with appropriate temperatures.

4.7 Choose the most appropriate tool for measuring

Time, money, and measurement must be on-going activities. Rather than teaching special units on these topics, teachers should include these skills and concepts in lessons throughout the year.

A. Show student a book. Ask child to get from the measurement center what is needed to determine how long the book is (an inch or centimeter ruler).

Ask child how to figure out what the book weighs. What is needed to weigh the book? Would a ruler help?

B. Ask student to measure the length of a pencil. Give student the choice of using a centimeter ruler or a meter stick (or an inch ruler or a yard stick).

C. Ask student which container would be better to use to measure the sugar for a cookie recipe.

D. Have student choose the best way to find out the number of juice cups which are in a pitcher.

4.8 Cover areas with non-standard units

A. Give student a workmat. Tell student to completely cover the mat with Unifix cubes. Could you cover this mat with playing cards? Would it take more or fewer playing cards?

B. Give student an outline and some pattern blocks. Do you think you could completely cover this shape with blue parallelograms? With yellow hexagons? Would you use more parallelograms or hexagons?"

C. Using a non-standard unit such as paper squares or playing cards, have the student estimate, then measure the surface of a book, the desk top, the teacher's desk top, or the top of a game board.

D. Have student trace the shape of a cookie cutter on paper and fill the shape with beans. How many beans does it take to fill the shape? As a more advanced activity, have child trace cookie cutter on centimeter grid paper. Count the squares. (Have student estimate, combining partial squares to make whole units.)

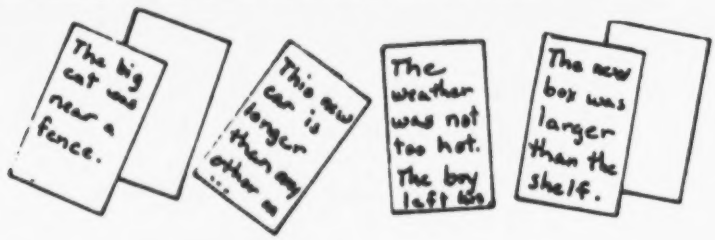
4.9 Use measurement related words in daily vocabulary

A. Have student correct silly sentences (oral or written task). For example: "John is getting very fat. He weighs 60 inches." "Tonya poured juice for

each student. She gave each child a quart." "Susan walked a long way. She walked 10 centimeters." "The bug crawled across the desk. It went about six miles."

B. Observe if student uses appropriate measurement words in describing activities. For example, does the student say that the pencil is about five inches long or does the student state that the pencil is five?

C. Using pictures cut from magazines or drawings, the student correctly matches the picture card to the sentence card.



4.10 Sequence months; use calendar to solve problems

A. Show child cards with the months of the year in order. Turn some months over and ask "What's missing? What month comes before June, after February, between August and October? How many months are in a year? What month will it be two months from now?"

B. Give cards with the names of the months to 12 children and ask them to put themselves in order.

C. Give one student cards with the months of the year to put in order.

D. Divide the children into two teams. At a signal, children must put themselves in order by their birthdays beginning with January and ending with December. Extend the activity to make a birthday graph. Determine which month has the most birthdays or how long until a certain child's birthday.

E. Group the months of the year by the season. Explain why your group might be different from another student's. Which pictures go with winter? with spring? with summer? with fall?

4.11 Tell time to nearest half hour

A. Give student a paper clock with movable hands. Ask child to show the times that special activities take place during the day. Use hour and half hour designations such as "We begin school at 8:30" or "We go to the library at 10:30."

B. Show student a digital clock. *Note problem solving activity in objective 5.5, part D.* Ask child to position the hands on the circular clock to show the same time. Reverse the activity by showing student a circular clock and asking the child to make the same time on the digital clock.

C. Given pictures of clocks, have student tell the times. Relate circular clock times to digital clocks.

D. Given pictures of clocks with no hands, have student draw hands to show designated times. Write corresponding digital times.

4.12 Give values of sets of coins; identify coins needed to buy items

At second grade level the goal is for students to use pennies, nickel, dimes, and quarters.

A. Ask student to identify coins and give value of each coin.

B. Show student a group of coins. Ask child how much money there is.

C. Student is told an amount of money (for example, 57¢). Have student show the amount with real coins, play money, coin rubbings, or money stamps.

D. Student can match a set of coins to an equivalent set such as a quarter matches two dimes and a nickel or three dimes and a nickel match a quarter, one nickel, and five pennies.

E. Show student toy items with price tags. Student gives the correct amount of money to purchase the toy. Ask the student to show you a different group of coins to make the same purchase.

F. Given a worksheet, have student tell value of a set of coins. Or, have student circle a given amount of money.

4.13 Estimate costs; make change using coins

A. Tell the student you would like to spend between twenty and thirty cents for two items. Ask student to name pairs of items you could buy. Repeat the activity asking for three items you could purchase for a given price. Have student explain the thinking in making the estimates.

B. Price the items in the classroom store with tags showing values less than a dollar. Have a wide variety of prices. Ask child to choose three items and estimate if they will cost more than fifty cents or less than fifty cents. Use a calculator to check the estimates.

C. Tell student that you have bought an item which costs twelve cents. You want to pay for the item with two dimes. Have student give you your change. Repeat with other appropriate costs and coins.

D. Observe students in the classroom store. Can the student offer appropriate coins (not necessarily the exact amounts) to the clerk to make purchases? If the student is the clerk, can the child give back the correct change?

E. Given a worksheet with amounts spent and money rendered, have student indicate coins given in change. *This is very difficult for most students. Because the objective is to use money in real life situations (i.e. making change), this should not become a written subtraction problem.*

4.14 Read/ write money expressions (cent notation)

A. Read amounts of money written with cent mark. Read money amounts written in decimal form. Given an advertisement, have student read appropriate sale prices.

B. When student determines the value of a set of coins, ask child to write how much money there is using cent mark.

C. After student has worked with place value to hundreds, ask student to write value of set of coins using dollar and decimal signs.

4.15 Solve simple time/ money problems

A. Show student a quarter, two dimes, a nickel, and three pennies. Do you have enough money to buy a toy that costs 45¢? Would you have any money left over? If so, how much?

B. Tell the student that you want to buy an ice cream cone with three scoops. Each scoop costs ten cents. If you have 40¢, could you buy a three-scoop cone? If each scoop costs five cents, how many single cones could you buy for 25¢?

C. Ask a student to show you all the ways to make 30¢ using ten coins or fewer.

D. Letecia has 12¢. She wants to buy stamps. The store has stamps which cost two cents, stamps which cost three cents, and stamps which cost six cents. What are some of the possible ways she could spend her money?

E. Have children put their heads down and take a "minute nap." Have them put their hands up as soon as they think a minute has passed. Children can check their own estimates by observing if the teddy bear (held by the teacher) has awakened.

F. Show the student 3:00 on the clock. Move the hands to show 4:00 and ask how much time has passed. Repeat activity with 7:00 and 7:30. How much time has passed?

Competency Goal 5: The learner will use mathematical reasoning and apply problem solving strategies.

5.1 Use calculator to solve problems

Technology is changing the manner in which mathematics is applied in everyday consumer situations. Not only calculators and computers but also video equipment, tape recorders, and other electronic devices are impacting society. It is the school's responsibility to be certain that all children learn to use these tools appropriately. Computer programs provide motivating drill but more importantly offer problem solving applications for students at all elementary levels. Logo should be explored by second grade students both as geometry lessons and as problem solving lessons. Children wear calculators on their wrists and tend to believe whatever magically appears on the display. Schools must help the students understand that calculators function because people push certain buttons, not because the machines can"think".

A. Ask child to use calculator to find sums and differences. For example, 28 + 34 or 76 - 21.

B. Have student use calculator to solve problems in textbook. *Be certain to provide problems where calculator is beneficial. Some problems are easier to solve mentally than with a calculator, and students should realize this from earliest lessons. Problems with 3-digit numbers or several addends are appropriate.*

C. Observe if child uses calculator correctly to tabulate information gathered in experiments or surveys.

D. Have student use calculator to model skip counting. *When students are comfortable using the constant for addition or subtraction, they will be able to explore problems such as this: Count on by 2's starting at 24. If you enter 24 + 2 and press "=" six times, what number do you think you will see?*

5.2 Solve spatial visualization problems; demonstrate visual memory

Being able to bubble in answers accurately on a separate score sheet is a spatial problem for many children. A student's ability to transfer information is not necessarily an indication of the student's understanding of the content. Hopefully there will be a carry-over from spatial visualization activities to real-life applications such as using computer-scored sheets, judging distances when riding a bike, and eventually parking a car.

A. Have student solve jigsaw puzzles. Have students create puzzles by cutting up greeting cards into puzzle pieces and storing the homemade

puzzles in sandwich bags.

B. Have student fill in pattern block designs in more than one way. Record the blocks used each time.

C. Show student a hidden shapes picture. Have child identify as many figures as possible. *Pictures such as those in Highlights magazine or abstract designs similar to the sample in the grade one strategies are both appropriate.*

D. Arrange six to eight pattern blocks in a certain order (not a recognizable pattern). After the student examines the arrangement, scramble the blocks while the student's eyes are closed. Ask child to arrange the blocks into the original order.

E. Show student a drawing for a brief period. Ask the child to draw the figure from memory. For a group activity, place a design on the overhead for a few seconds. Ask students to draw what they remember.

F. Ask student to draw a picture of the front of the school from memory, making the picture have as much detail as possible.

G. Ask student to draw a floor plan of the bathroom at home. Draw a plan of the kitchen, etc.

H. Have student make a design on a geoboard. Ask student to record the geoboard design on dot paper. Reverse the process: give student a design drawn on dot paper and have the child make it on the geoboard.

I. Give student a picture drawn on dot paper or on grid paper. Have child make an enlarged picture by drawing on dot or grid paper which uses larger units.

J. Given a picture of a block design (not actual size of the blocks), have student build the design.

K. Use the seven tangram pieces to make a square. Use tangrams to complete other puzzles.

L. Give one student a design card. Ask the student to describe the design to another person. The second child tries to draw the design from the description.

M. Flash a design on the overhead. Have student draw the design from memory. If necessary, give student a second look.

N. Make puzzle outlines with two pattern block pieces or shapes cut from stiff paper. Draw other puzzles which are similar but which cannot be completed with the two shapes. Ask student to identify which puzzles could be completed with the two blocks.

5.3 Make reasonable estimates

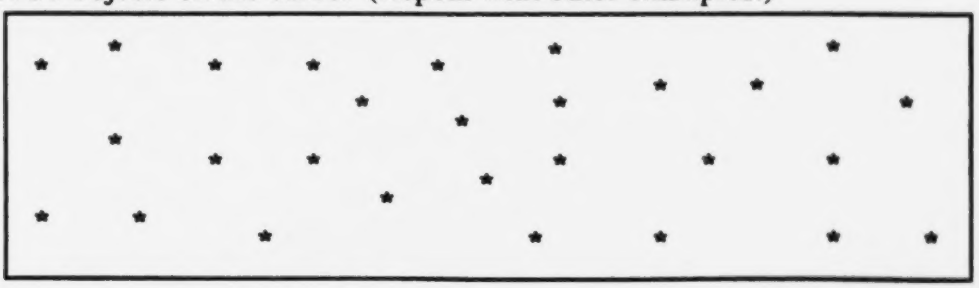
A. Ask child which of these tasks could be done in an hour or less:

- Wash the dishes
- Count to 100 twice
- Sing a song
- Walk around the school 3 times
- Get a night's sleep
- Travel to California

B. Ask student how many cups of juice can be poured from this can. About how many name tags like a sample can be cut from this piece of construction paper?

C. Ask student if there are enough chairs in the classroom for the students and their parents to be seated at the same time. Would there be enough napkins in this pack to give one to all of the students?

D. Show student cards with dots or stars. Ask child if there are closer to 20 or 50 objects on the cards. (Repeat with other examples.)



E. Ask student to estimate about how many times students could touch their toes in one minute. Have students experiment. (Combine this with a graphing lesson.)

5.4 Describe processes in finding solutions; suggest alternate strategies

A. Given a measurement task (for example, to find which of three objects is the heaviest), ask child to describe how to complete the assignment.

B. Give student an incomplete list of steps in completing a task. Have student tell what steps are missing.

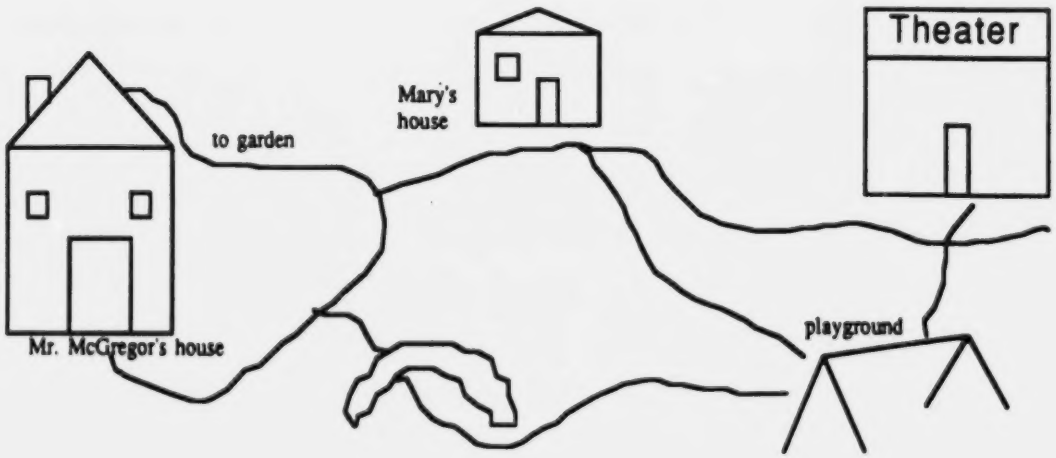
C. Tell student there is something of interest in a box. Ask student to explain different ways to find out what is in the box.

D. Given a problem, ask child to describe how the problem was solved. Following a group problem solving activity, ask student to tell (write) how the problem was described.

E. Given a page with problems already solved (some with mistakes), ask student to correct errors.

F. Give students a series of cartoon pictures. In the first two frames present an open-ended problem. Ask student to write two possible endings to the story.

G. Given a map, ask student to suggest several routes from Ray's house to the movies. Ask student if there is more than one way to go from home to the school.



H. Have student play Nim-type games such as "19 Nice Ones". Challenge students to find a way to always win. Game rules: Place 19 counters on a game mat. The two students take turns removing one, two, or three counters at a time. The object of the game is to force the opponent to pick up the last counter. *Game mats could be easily constructed to go with any topic currently being studied in the class. For example, if the science unit is on dinosaurs, draw 19 dinosaur eggs on which students would place their counters.*

5.5 Solve simple logic problems

A. Give student two kinds of crackers and two kinds of juice. How many different snacks could be created? Have student cut out paper doll clothes. Ask student how many different outfits could be made with a green shirt, a red shirt, blue pants, and brown pants. Have student draw or cut out enough shirts and pants to picture the solution.

B. Mary, Sam, and Tomeka brought teddy bears to class. Sam's bear does not have clothes. Tomeka's bear has more buttons than the others. Ask child which bear belongs to each child.

C. Without counting the set, with a partner figure out how many dots there are on a set of double six dominoes. Ask students to explain how they figured this out. *Be certain that students have had opportunities to work with dominoes in other lessons. Have students use dominoes to verify their answers.*

D. In a group, have students make digital clocks. What numbers need to go on each of the four paper strips? Are the numbers the same? Why?

E. Given a matrix, student will place markers correctly on the board. For example: Arrange four markers each of red, yellow, blue, and green on the grid so that no color is repeated in any row or column.

R	B	Y	G
G	Y	B	R
B	R	G	Y
Y	G	R	B

One possible solution

F. Given a logic puzzle, student will solve the problem. For example, Rhonda, Jason, and Ted brought their favorite toys to school. One student brought a ball and bat, one brought a bag of marbles, and one brought a plastic man. Ted enjoys playing with his friends even though his leg is in a cast. Jason, who does not like to play outside, has a collection of robots. Rhonda's nickname is "Speedy". Which toy belongs to which student?

5.6 Use models or act out to solve problems

A. Read a story problem to the group. Ask student to act out the problem. Ask a student to explain why acting out the story helps to find the solution.

B. Have student use Unifix cubes to solve problems:

"There are six cubes."

"Half of the cubes are yellow."

"One cube is red."

"The rest of the cubes are blue."

"How many cubes are blue?"

C. Give student a container with goldfish or animal crackers. Tell story problem and have student model the problem.

D. Observe if student uses models when solving problems independently.

5.7 Use drawings/ diagrams/ organized lists to solve problems

A. Give student a problem that involves finding all possibilities. Observe how the student finds the solution. For example, "There are four girls in the tournament. How many possible two people teams could be set up? The girls - Sue, Jill, Maria, and Keiko - all work well together."

B. Ask student to make a chart to show what possible combinations of stamps you could purchase from the machine that holds 2¢ stamps, 3¢ stamps, and 5¢ stamps if you have only 12¢.

C. Ask student to use information on a table or graph to answer questions. For example, a table in science or social studies could serve as a source of data.

D. Have student "bury" a treasure somewhere in the classroom. Ask child to write directions for finding the treasure in five or more instructions.

E. Give student series of clues to find a number on the hundred board. Ask student to write other "number stumpers". For example:

"The number has 2 digits.
The digits have a sum of 10.
The number is less than 30.
The number in the ones place is 9.
What is my number?"

F. Have child display information from a class experiment and create (answer) questions about the data.

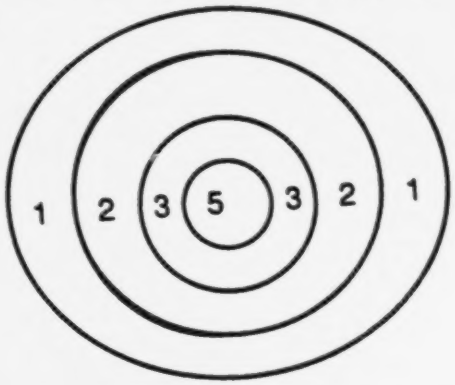
G. Give student three digits (for example, 3, 6, 7). How many different two-digit numbers can be made with these digits.

5.8 Use guess and check to solve problems

A. Give student a large container of popcorn, beans, or rice. Do you have enough to give each of ten children a cup?

B. Show student items from the class store with price tags attached (or pictures of items). Ask child which three things could be purchased for a given amount. *(See Objective 4.13)*

C. Ask student where Joy's three arrows might have hit if her score was seven. Where could her three arrows have hit if her score was six?



D. Give student four number cards and ask student to arrange them to make the largest sum, smallest sum, largest difference, or smallest difference.

Competency Goal 6: The learner will demonstrate an understanding of data collection, display, and interpretation

6.1 Collect, sort, organize, display information

Students collect, organize, and display information in order to answer questions and to help see the relationships within data. They learn to summarize as well as answer factual questions. They learn to use their data to make predictions. Because we live in a world of information, students must have experiences that help them to evaluate that information and to decide about the usefulness of data. Graphing should not be taught as a separate unit but interwoven throughout the year; graphing skills are useful in all content areas and are among the most important real applications of mathematics in early grades.

A. Ask student to survey classmates on a given question. Have student talk to at least 12 people. "What were the results of the poll? If you were to ask 12 students in the 5th grade the same question, do you think the results would be the same?"

B. Have entire class fill out a brief information sheet with basic information about themselves. Give forms to a group of students and ask them to organize them so that they could make a display of who has birthdays each month. *Forms can be used for many activities since they ask for basic information which all students need to know. Completing the form might be a part of a reading lesson and using the data a part of math lessons. A sample form is included. Using the form, students could make charts about brothers and sisters or about pets, for example.*

C. Conduct a taste test and graph the results. Use three different samples of the same food (for example, chocolate chip cookies or corn chips). Do not allow students to see the original containers until after the experiment. Have children agree on criteria for judging. Students should write sentences to explain the experiment and the results.

D. Give student the change from your wallet. Ask child to make a graph showing the coins you have. Ask student to write five questions for another child to answer about the graph.

6.2 Summarize/ interpret information on charts/ graphs; make predictions

A. Have class graph the results of a survey such as this: "If you could choose, would you be younger, the same age, or older than you are?" Ask student to explain the results of the survey (how most people voted, etc.) Ask interpretation and opinion questions also: "Why do you think most students said they wanted to be _____? If you asked people who are the teachers' ages, do you think the results of the survey would be the same?"

B. Given a graph from the textbook, ask student to explain the information.

C. Have student make a display with information from the cafeteria about students' favorite meals. Ask students if they think the students in their classroom would agree with the choices. Have students decide how to gather and compare information. Ask children to talk about what they learned from the two sets of information. Which lunches are the most popular? Why?

6.3 Collect and display data over a period of time

A. Have student place information on a graph to record daily attendance, temperature each day at noon, weather each day at noon (children decide categories), number of books checked out of the library by students in the class, etc.

B. Have student record growth of a plant (or bean sprouts) each Monday, Wednesday, and Friday for several weeks. Draw a picture and describe what is observed.

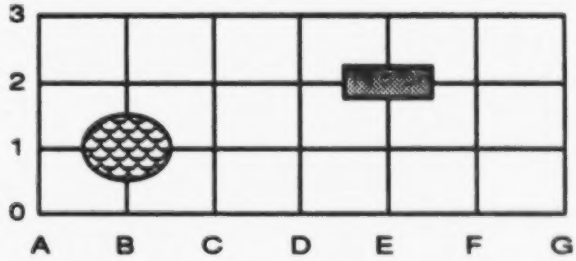
C. Ask students if they think more children are absent on Fridays than any other days. Check attendance records by days for six weeks. Explain.

6.4 Locate points on number line; explore positions on grids

A. Given a number line, have student identify a labeled point. Given a number line with some labels missing, have student identify missing numbers.

B. Given a number line labeled with 2's or with 5's or with 10's, have student identify an unnamed point. For example, "Where would 3 be on this number line? Where would 7 be on the number line?"

C. Place toys at intersections on a large graphing mat. Have students describe where items are.



D. Use knowledge of grids to read simple maps.

6.5 Complete simple experiments; describe results; make predictions

A. Give student various models of solids. Ask child to predict which ones will roll straight down a ramp and which ones either will not roll or will roll crooked. Have student experiment and record. Ask student to describe what happened. Based on the experiment ask the student to predict whether other solids will roll straight. *Use wooden blocks and plastic solid models as well as cans, cups, cones, cylinders and cubes from home.*

B. Give student an eye dropper and a cup of water. Ask student to predict how many drops of water will go on a nickel, on a dime, and on a penny before the water spills off. Ask child if there is a way to do the experiment so that after checking one coin the student will be able to make improved predictions about the others. *For example, a student might begin with the smallest coin and revise predictions for the penny and nickel after*

counting drops on a dime. This is a good cooperative learning lesson. Divide class into groups of 3 or 4 students and compare results obtained by each group. Ask student if the experiment were repeated, what might the number of drops be for each coin. Why? What would happen if soap were added to the water and the experiment repeated?

C. Ask student, "What numbers could be rolled on a die? Are there any other possibilities?" Have student roll the die 20 times and record what number shows each time. *Making a chart and keeping a record such as this is difficult for students. For many children this activity will be appropriate only as a group lesson.* "Tell about the experiment. If you rolled the die 20 more times, do you think you would have the same record? Why? Would you like to try this again?"

D. "How many jumping jacks can most people do in one minute?" Ask student to plan an experiment, carry it out, and report the results.

Competency Goal 7: The learner will be able to compute using whole numbers.

7.1 Use manipulatives/ counting strategies to derive addition/ subtraction facts; write equations

The process of learning number facts is a long one which ideally utilizes children's different learning styles as well as builds upon what students already know. Studying "fact families", learning facts by counting on, making ten or other grouping strategies, and drilling on a few facts at the time are all ways to help children master the facts. Pressuring children to not count on their fingers simply makes them go "underground." Hands disappear under the desk or press the knees as students continue to count with their fingers. When students are involved in doing mathematics, counting items, grouping objects into tens and ones, measuring, creating graphs, counting money, and building geometric models, they are experiencing numbers and learning the facts informally. Games in which children drill using manipulatives and eventually those which encourage quick recall help make practicing number facts more enjoyable. Encourage students to explain their thinking and to be efficient in the strategies they use for solving individual facts. Praise them for their accomplishments, and do not be in a hurry to take the manipulatives away.

A. Given a series of number facts, ask student to explain ways to figure them out. (i.e., What strategies would help to solve these problems?) Encourage student to "think out loud" to determine if child is using any strategies to make solving the fact easier.

B. Ask student to explain why adding or subtracting "one" or "two" can be figured out in one's head.

C. Have student "act out" number facts which are efficiently solved by counting on. Have student stand on a walk-on number line and demonstrate the solution to the number fact. For example, to find 9 + 4 the student steps to the nine and then advances four units more to land on 13. A desk top number line is best utilized if the student has a marker (teddy bear counter, toy dinosaur, car eraser, etc.) to physically model the fact. *Notice that subtraction on a number line is a different model from subtraction that is illustrated by creating a set and removing part of it.*

D. Have students play "Top Sum" using one die with numerals and one with dots to encourage students to count on as opposed to counting all. A score sheet with rules for "Top Sum" is included in this notebook.

E. Given a worksheet, ask student to circle all facts already memorized and write the sums or differences. Have students find remaining facts using counters.

7.2 Use addition or subtraction to solve problems

This objective is directly related to all of the objectives in competency goal 5. As those are evaluated, this objective is also. Students traditionally are not as good at solving word problems as we would like for them to be. The number of word problems students are asked to do is small compared with the amount of computation drill. As the amount of time devoted to problem solving is increased, efforts should be made to include a wide variety of applications in addition to routine word problems.

A. Give student a number fact (6 + 5 or 12 - 7, for example). Ask child to write a story to go with the problem and illustrate the story. Feature student's work on the "Problem of the Day" board.

B. Give student a cartoon picture. Have child write a story about the picture and pose an addition or subtraction problem in the story. Ask student to write the solution on the back. Display student's work.

C. Given a worksheet, student will read story problems and solve correctly.

7.3 Tell missing addends for addition facts

A. Using facts with which the student has demonstrated memorization, ask student to tell missing addends. For example, three plus what number equals six? What number plus six equals eight?

B. On a worksheet student fills in missing numbers for facts that have been memorized.

7.4 Model/ record/ solve 3 single digit addition

A. Place six Unifix cubes of three different colors (total of 18 counters) in a bag. Have student reach into the bag and remove a handful of cubes. Sort by color and write the number sentence. Repeat, recording the problem vertically if student first wrote horizontally (or vice versa).

B. Roll three dice and add. Write the number sentence to show what has taken place.

C. Draw three numeral cards. Use counters in three colors to model a problem which could be created with the numbers. Write the number sentence.

D. Given a worksheet, ask student to model problems and find solutions. Given a worksheet, ask student to solve problems.

7.5 Model/ record/ explain 2-digit addition/ subtraction

When students have many experiences with building numbers, adding to that set, and taking cubes away from the set, and when they have worked with naming numbers in different ways (for example, 3 tens and 4 ones is the same value as 2 tens and 14 ones), problems with regrouping (renaming) are not mysterious and do not present the major difficulties they have in the past with students who have few experiences with place value. By spending many weeks building and recording problems (i.e. work at a concrete level), children can be introduced to 2-digit operations with and without regrouping simultaneously.

A. Make models of numbers on five different place value mats. Ask student to tell about each set. Have student choose any two groups and explain what the sum would be if they are added. *Create models so that some problems would involve regrouping and others would not. If students have had many experiences renaming numbers in numeration activities (competency goal 1), regrouping at a concrete level should not be difficult.*

B. Ask student to build a number greater than 30 but less than 50. Have student subtract the number of children in the class.

C. Direct student to build a two-digit number on the workmat. Have

student build a second number. Combine the sets, trading ones for a ten if necessary, and explain the sum. Using that sum, ask student to remove a given amount and explain the results. For example:

"Build 34.
Build 23.
What is the sum? (57)
From your 57, take away 14.
What do you have now?"

D. Give two students a place value mat each and a set of bean sticks. Have students play with a file folder separating their workmats so they cannot see what the other is building. Rules: Each student draws a card and builds a two-digit number on the place value mat and predicts if the sum of the two numbers being built will be greater or less than 50. Students combine sets. If one or both students predicted correctly, they win a point. The first player to get ten points wins. *Cards should designate any numbers from 10 to 40. A deck of 20 cards, shuffled after each turn, is sufficient.*

E. Given the numbers, student models a two-digit addition problem. The student then records what has taken place and explains the solution. *The importance of continuing language experiences with the math lessons cannot be over-emphasized.*

F. Given the numbers, student models a two-digit subtraction problem. The student then records what has taken place and explains the solution.

7.6 Demonstrate ways to verify reasonableness of answers

A. Student uses estimation skills to approximate a sum or difference. For example, given 27 + 41, the student explains that 27 is almost 30 and 41 is about 40, so the answer should be close to 70. *Students' explanations will demonstrate levels of understanding.*

B. Student can verify the answers by using Unifix cubes or other manipulatives to model the processes of addition, subtraction, or

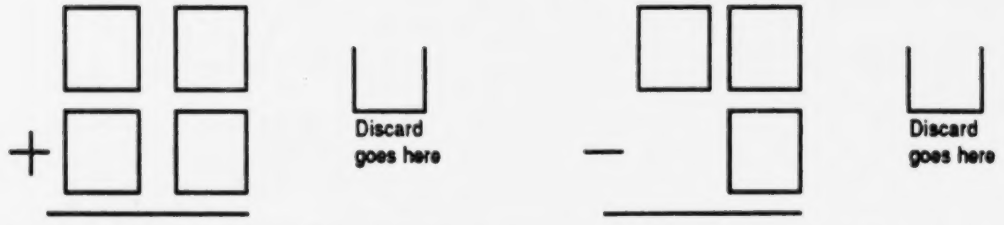
nonroutine problems.

7.7 Add/ subtract 2-digit problems with/ without renaming, using manipulatives as needed for accuracy

A. Given a workbook page the student can correctly solve two-digit problems. Students must be encouraged to use manipulatives if they cannot successfully explain the processes or solve the problems correctly.

Students should not treat two-digit exercises as separate number facts. For example, 47 + 32 is not two separate facts of 7 + 2 and 4 + 3. Finding sums in this manner leads to solutions such as 28 + 34 = 512.

B. Student can successfully play games involving two-digit addition and subtraction. For example, play Foxy Boxes. Rules: (1) Student draws "gameboard" with a discard box (see example below). (2) The teacher will call out five digits. As the digits are called out, the student must write one number in each spot on the gameboard. Once a digit has been written, it cannot be moved. (3) The object of the game is to build the highest sum. (4) To play a subtraction game the gameboard will have places for only four digits. The greatest difference wins.



FOXY BOXES: Gameboard for addition version Gameboard for subtraction version

7.8 Add/ subtract 1 or 10 mentally

A. Given a number, the student will tell what the results are when one is added or subtracted.

B. Given a number, the student will tell what the results are when ten is

added or subtracted.

C. Using a hundred board play "Arrow Math" games *(for more advanced students)*. For example, "Begin on 34. Move marker one space to the right, move one more space to the right, move down one. What number are you on?" *(46)*

7.9 Model repeated addition

A. The teacher directs the student to be a farmer who is planting rows of corn. He wants four rows with three kernels of corn in each row. Using Unifix cubes, the student models the rows of corn. Ask student how to figure the total number of kernels. *(Counting, repeated addition, and multiplication.)*

B. Ask student to find the number of legs on four chairs. Ask student, how can this be determined without counting? *(4 + 4 + 4)*

C. Give student circles of yarn and counters. Ask child to show three sets of five. Repeat with other numbers.

D. Using Unifix cubes, ask student to model examples of repeated addition (for example, crayons in two boxes, wheels on three tricycles, etc).

E. Given word problems using repeated addition *(simple multiplication)*, have student solve the problem using manipulatives.

7.10 Model sharing equally

A. The student is given beans (or any counters) and asked to share equally with three friends. "Left-overs" are returned to the teacher.

B. Given a quantity of beans and cups, the student distributes the beans equally in the cups. Remaining beans are left on the desk.

C. Ask student to deal cards for a game, making certain that each player has the same number.

7.11 Memorize addition/ subtraction facts

An addition table, highlighting easier facts, is included in the notebook.

A. Given an easier addition fact, the child can answer without counting or hesitating. Given an easier addition fact, the student can write the sum immediately.

B. Given an easier subtraction fact, the student can respond without hesitating. Given an easier subtraction fact, the student can write the difference immediately.

C. Given any addition fact, the student can respond orally or in writing without hesitation.

D. Given any subtraction fact, the student can respond orally or in writing without hesitation.

E. Given a worksheet with addition and subtraction facts, the students can write answers quickly.

F. The student applies knowledge of addition and subtraction facts in problem situations.

Top Sum

	Name:	Name:	Name:	Name:
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				
13.				
14.				

Rules:

1. Two, three or four students may play. Students write their names in the boxes.
2. At a turn each student rolls 2 dice (one regular die and one with 9,8,8,7,7,6).
3. As students roll the dice, they add and record the number sentences. Circle the winner of each round. The student with the most "top sums" is the winner.

STUDENT INFORMATION SHEET

Your name _____
First Last

Your address _____

Your town _____

Your state _____

Your telephone number _____

Your birthday _____

How do you usually come to school? (Check one space.)

_____ car _____ bus _____ walk

Do you have brothers? _____ If yes, how many? _____
(yes or no)

Do you have sisters? _____ If yes, how many? _____
(yes or no)

Do you have a pet? _____ If yes, tell about your pets.
(yes or no)

_____

_____

ADDITION TABLE

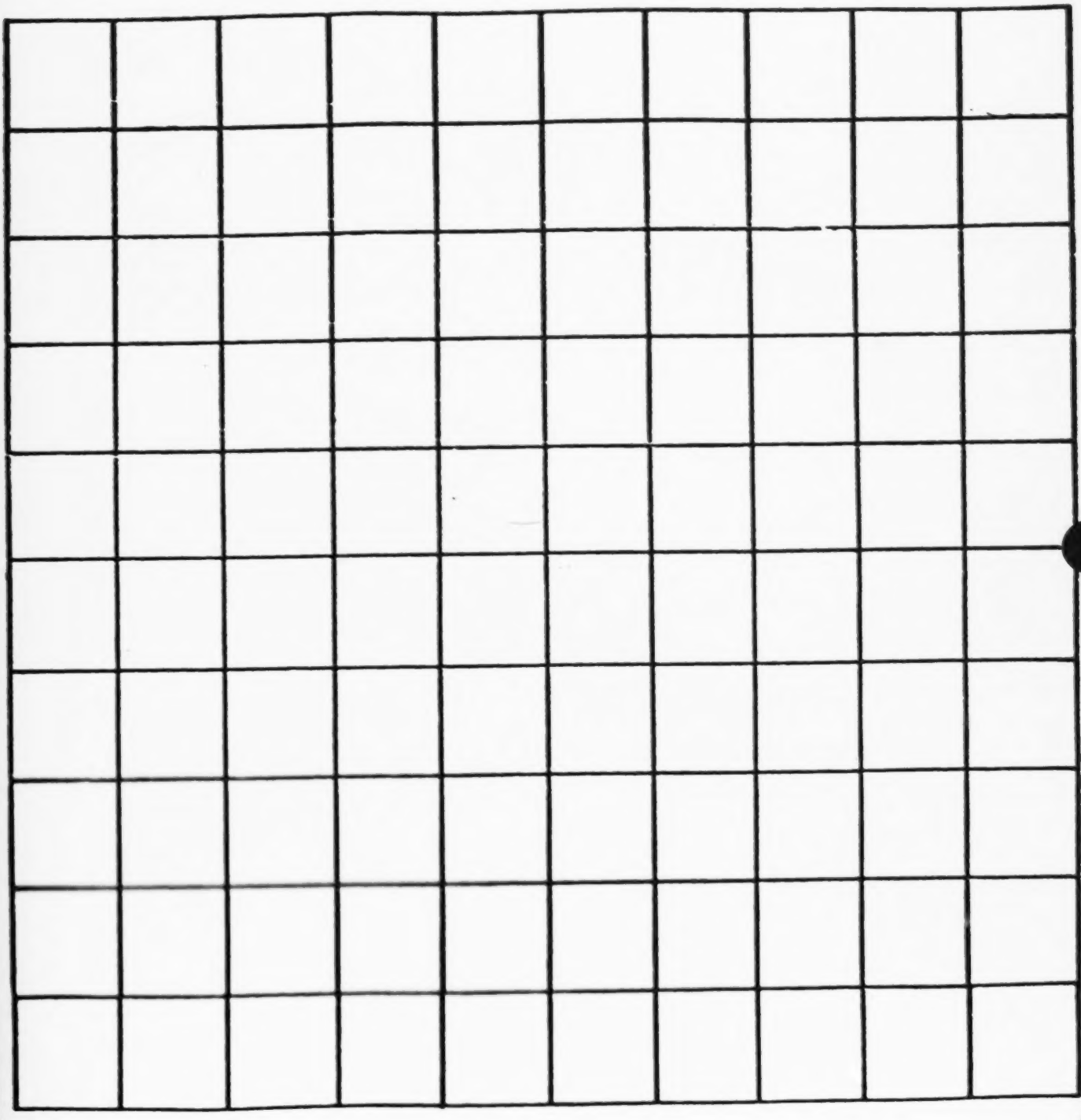
+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Easier Facts

Medium Facts

Hard Facts

B-57



Place Value Mat

tens	ones
135	136

Place Value Mat

hundreds	tens	ones

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Appendix

GRADE 1: COMPETENCY GOALS AND OBJECTIVES

Competency Goal 1: The learner will identify and use whole numbers, 0 to 100.

1.1 Count using 1 to 1 correspondence

1.2 Make sets; Match numerals

1.3 Compare and order sets

1.4 Identify ordinal position

1.5 Conserve numbers

1.6 Read and write numerals; Read number words 0 to 10

1.7 Count on

1.8 Recognize one more/ less/ before/ after/ between

1.9 Compare/ sequence numerals

1.10 Rote count by 1's, 10's, 5's and 2's; Group objects to count by 10's, 5's, and 2's

1.11 Count by using tallying

1.12 Make reasonable estimates of "how many"

1.13 Group objects into tens and ones; Record

1.14 Recognize models; Build 2-digit numbers; Write numerals

1.15 Represent numbers in a variety of ways

Competency Goal 2: The learner will identify and use geometric ideas.

2.1 Identify open and closed figures

2.2 Identify, describe, and model plane figures; Recognize in environment

2.3 Use directional/ positional words

2.4 Describe likenesses and differences

2.5 Identify and describe solid figures; Recognize in the environment

continued on next page

Competency Goal 3: The learner will demonstrate an understanding of classification, patterns, and seriation.

3.1 Describe objects by their attributes; Compare and order

3.2 Sort by given attribute/ by more than one attribute; Explain sorting rules

3.3 Sort objects by own rule; Explain sorting rule

3.4 Copy/ continue patterns; Translate into different forms

3.5 Create patterns with actions/ words/ objects

3.6 Find and correct errors in patterns

3.7 Identify patterns in the environment

Competency Goal 4: The learner will exhibit skills in using measurement.

4.1 Compare objects/ use appropriate vocabulary

4.2 Use non-standard units to measure length, weight, capacity

4.3 Identify equal/ unequal parts

4.4 Use time related words in daily vocabulary

4.5 Name/ order days of week; Name months of year

4.6 Use information on a calendar

4.7 Tell time to hour (face/ digital clocks)

4.8 Identify/ give values of penny, nickel, dime

4.9 Identify coins needed to buy items; Make different sets with same value

continued on next page

Competency Goal 5: The learner will use mathematical reasoning and apply problem solving strategies.

5.1. Use calculator to solve problems

5.2 Use visual memory; Solve spatial visualization puzzles; Copy simple designs

5.3 Estimate/ suggest reasonable solutions to problems

5.4 Use models or "act out" to solve problem

5.5 Use drawing/ diagrams to solve problems

5.6 Use guess and check to solve problems

Competency Goal 6: The learner will demonstrate an understanding of data collection, display, and interpretation.

6.1 Gather, organize, and display information as a group activity

6.2 Answer questions about charts/ graphs

6.3 Make predictions based on experiences

Competency Goal 7: The learner will compute using whole numbers.

7.1 Model concept of addition

7.2 Explore combinations for numbers to 10

7.3 Model subtraction as take-away; As comparison

7.4 Create and solve problems using addition/ subtraction

7.5 Relate addition/ subtraction to symbolic notation/ write equations

7.6 Use counting strategies to find sums/ differences

7.7 Memorize easy addition facts

7.8 Memorize easy subtraction facts

7.9 Model 3 single digit addtion/ record

7.10 Model 10 more/ less

7.11 Model 2-digit addition/subtraction with multiples of 10

GRADE 2: COMPETENCY GOALS AND OBJECTIVES

COMPETENCY GOAL 1: The learner will identify and use numbers to 100 and beyond.

1.1 Rote count beyond 100

1.2 Estimate; Use various counting strategies to determine how many

1.3 Group tens and ones; Read/ write standard numerals

1.4 Identify and use 10 more and 10 less

1.5 Compare and order numbers; Identify missing numbers in a sequence

1.6 Read/ write word names for numerals correctly

1.7 Skip count by 2's, 5's, 10's; Group objects to count by 3's and 4's

1.8 Name nearest multiple of 10 on a number line when given a number

1.9 Identify odd and even numbers using objects

1.10 Divide regions/ sets into halves

1.11 Group objects into ones, tens, and hundreds

1.12 Model 3-digit numbers; Choose/ read/ write correct numerals

Competency Goal 2: The learner will demonstrate an understanding and use of geometry.

2.1 Identify/ make plane & solid figures, including cylinders, using appropriate vocabulary

2.2 Identify and make figures with line symmetry

2.3 Match congruent figures

2.4 Replicate 3-dimensional designs using models

2.5 Recognize square corners/ geometric figures in the environment

continued on next page

Competency Goal 3: The learner will demonstrate an understanding of classification, pattern, and seriation.

3.1 Describe similarities and differences

3.2 Classify by more than one attribute; Describe rules used in sorting

3.3 Define and continue patterns; Translate into different forms

3.4 Identify classification and patterning in the environment

3.5 Identify/ correct errors in patterns

3.6 Use patterns to continue numerical sequences

3.7 Order objects and events; Use ordinal numbers

Competency Goal 4: The learner will understand and use standard units of metric and customary measurement

4.1 Estimate one inch/ centimeter; Measure length to nearest inch/ centimeter

4.2 Measure capacity to nearest cup

4.3 Estimate number of smaller units contained in larger unit

4.4 Weigh objects to nearest pound/ kilogram

4.5 Explore conservation of length, capacity, weight

4.6 Read Celsius and Fahrenheit thermometers

4.7 Choose appropriate tool for measuring

4.8 Cover areas with non-standard units

4.9 Use measurement related words in daily vocabulary

4.10 Sequence months; Use calendar to solve problems

4.11 Tell time to nearest half hour

4.12 Give value of sets of coins; Identify coins needed to buy items

4.13 Estimate costs; Make change using coins

4.14 Read/ write money expressions (cent notation)

4.15 Solve simple time/ money problems

continued on next page

Competency Goal 5: The learner will use mathematical reasoning and apply problem solving strategies.

5.1 Use calculator to solve problems

5.2 Solve spatial visualization problems; Demonstrate visual memory

5.3 Make reasonable estimates

5.4 Describe processes in finding solutions; Suggest alternate strategies

5.5 Solve simple logic problems

5.6 Use models or "act out" to solve problems

5.7 Use drawings/ diagrams/ organized lists to solve problems

5.8 Use guess and check to solve problems

Competency Goal 6: The learner will demonstrate an understanding of data collection, display, and interpretation.

6.1 Collect, sort, organize, display information

6.2 Summarize/ interpret information on charts/ graphs; Make predictions

6.3 Collect and display data over a period of time

6.4 Locate points on number line; Explore positions on grids

6.5 Complete simple experiments; Describe results; Make predictions

Competency Goal 7: The learner will be able to compute with whole numbers.

7.1 Use manipulatives/ counting strategies to derive addition/ subtraction facts; Write equations

7.2 Use addition/ subtraction to solve problems

7.3 Tell missing addends for addition facts

7.4 Model/ record/ solve 3 single-digit addition

continued on next page

7.5 Model/ record/ explain 2-digit addition/ subtraction

7.6 Demonstrate ways to verify reasonableness of answers

7.7 Add/ subtract 2-digit problems with/ without renaming, using manipulatives as needed for accuracy

7.8 Add/ subtract 1 or 10 mentally

7.9 Model repeated addition

7.10 Model sharing equally

7.11 Memorize addition/ subtraction facts

Schools have traditionally operated on the premise that practice is how most children learn new tasks. If this theory is true, it is important that children be given practice at which they will be successful. Independent practice, especially at abstract levels when students have no way of verifying results, must be delayed until the teacher is confident that the majority of the practice will result in correct responses by the students.

MATHEMATICS: Grade 1

Student_____

Teacher_____ School_____ 19__-__

Goal 1: The learner will identify and use numbers, 0 to 100.

1.1 Count using 1 to 1 correspondence				
1.2 Make sets; Match numerals				
1.3 Compare and order sets				
1.4 Identify ordinal position				
1.5 Conserve numbers				
1.6 Read and write numerals; Read number words 0 to 10				
1.7 Count on				
1.8 Recognize one more/ less/ before/ after/ between				
1.9 Compare/sequence numerals				
1.10 Rote count by 1's, 10's, 5's, and 2's; Group objects to count by 10's, 5's, and 2's				
1.11 Count by using tallying				
1.12 Make reasonable estimates of "how many"				
1.13 Group objects into tens and ones; Record				
1.14 Recognize models; Build 2 digit numbers; Write numerals				
1.15 Represent numbers in a variety of ways				

Code: M = Most of the time S = Sometimes N = Not yet

Goal 2: The learner will identify and use geometric ideas.

2.1 Identify open and closed figures				
2.2 Identify, describe, and model plane figures; recognize in environment				
2.3 Use directional/positional words				
2.4 Describe likenesses and differences				
2.5 Identify and describe solid figures; recognize in the environment				

Goal 3: The learner will demonstrate an understanding of classification, patterns, and seriation.

3.1 Describe objects by their attributes; Compare and order				
3.2 Sort by given attribute; by more than one attribute; explain sorting rules				
3.3 Sort objects by own rule; explain sorting rule				
3.4 Copy/ continue patterns; translate into different forms				
3.5 Create patterns with actions/words/objects				
3.6 Find and correct errors in patterns				
3.7 Identify patterns in the environment				

Code: *M = Most of the time* *S = Sometimes* *N = Not yet*

Goal 4: The learner will exhibit skill in using measurement.

4.1 Compare objects; Use appropriate vocabulary				
4.2 Use non-standard units to measure length, weight, capacity				
4.3 Identify equal/ unequal parts				
4.4 Use time related words in daily vocabulary				
4.5 Name/ order days of week; name months of year				
4.6 Use information on a calendar				
4.7 Tell time to hour (face/ digital clocks)				
4.8 Identify/ give values of penny, nickel, dime				
4.9 Identify coins needed to buy items; make different sets with same value				

Goal 5: The learner will use mathematical reasoning and apply problem solving strategies.

5.1 Use calculator to solve problems				
5.2 Use visual memory;solve spatial visualization puzzles; copy simple designs				
5.3 Estimate/ suggest reasonable solutions to problems				
5.4 Use models or "act out" to solve problems				
5.5 Use drawings/ diagrams to solve problems				
5.6 Use guess and check to solve problems				

Goal 6: The learner will demonstrate an understanding of data collection, display, and interpretation.

6.1 Gather, organize, display information as a group activity				
6.2 Answer questions about charts/graphs				
6.3 Make predictions based on experiences				

Goal 7: The learner will compute using whole numbers.

7.1 Model concept of addition				
7.2 Explore combinations for numbers to 10				
7.3 Model subtraction as take-away; as comparison				
7.4 Create and solve problems using addition/subtraction				
7.5 Relate addition/subtraction to symbolic notation/ write equations				
7.6 Use counting strategies to find sums/differences				
7.7 Memorize easy addition facts				
7.8 Memorize easy subtraction facts				
7.9 Model 3 single digit addition/ record				
7.10 Model 10 more/less				
7.11 Model 2-digit addition/ subtraction with multiples of 10				

Code: M = Most of the time S = Sometimes N = Not yet

MATHEMATICS: Grade 2 Student_____

Teacher_____ School_____ 19__-__

Goal 1: The learner will identify and use numbers to 100 and beyond.

1.1 Rote count beyond 100				
1.2 Estimate; Use various counting strategies to determine how many				
1.3 Group tens and ones; Read/write standard numerals				
1.4 Identify and use 10 more and 10 less				
1.5 Compare and order numbers;Identify missing numbers in a sequence				
1.6 Read/write word names for numerals correctly				
1.7 Skip count by 2's, 5's, 10's; Group objects to count by 3's and 4's				
1.8 Name nearest multiple of 10 on a number line when given a number				
1.9 Identify odd and even numbers using objects				
1.10 Divide regions/ sets into halves				
1.11 Group objects into ones,tens, and hundreds				
1.12 Model 3-digit numbers; Choose/read/write correct numerals				

Code: *M = Most of the time* *S = Sometimes* *N = Not yet*

Goal 2: The learner will demonstrate an understanding and use of geometry.

2.1 Identify/make plane & solid figures, including cylinders, using appropriate vocabulary				
2.2 Identify and make figures with line symmetry				
2.3 Match congruent figures				
2.4 Replicate 3-dimensional designs using models				
2.5 Recognize square corners/ geometric figures in the environment				

Goal 3: The learner will demonstrate an understanding of classification, pattern, and seriation.

3.1 Describe similarities and differences				
3.2 Classify by more than one attribute; Describe rules used in sorting				
3.3 Define and continue patterns; Translate into different forms				
3.4 Identify classification and patterning in the environment				
3.5 Identify/ correct errors in patterns				
3.6 Use patterns to continue numerical sequences				
3.7 Order objects and events; Use ordinal numbers				

Goal 4: The learner will understand and use standard units of metric and customary measurement.

4.1 Estimate one inch/ centimeter; Measure length to nearest inch/ centimeter				
4.2 Measure capacity to nearest cup				
4.3 Estimate number of smaller units contained in larger unit				
4.4 Weigh objects to nearest pound/ kilogram				

Code: *M = Most of the time* *S = Sometimes* *N = Not yet*

4.5 Explore conservation of length, capacity, weight				
4.6 Read Celsius and Fahrenheit thermometers				
4.7 Choose appropriate tool for measuring				
4.8 Cover areas with non-standard units				
4.9 Use measurement related words in daily vocabulary				
4.10 Sequence months; Use calendar to solve problems				
4.11 Tell time to nearest half hour				
4.12 Give value of sets of coins; Identify coins needed to buy items				
4.13 Estimate costs; Make change using coins				
4.14 Read/ write money expressions (cent notation)				
4.15 Solve simple time/ money problems				

Goal 5: The learner will use mathematical reasoning and apply problem solving strategies.

5.1 Use calculator to solve problems				
5.2 Solve spatial visualization problems; Demonstrate visual memory				
5.3 Make reasonable estimates				
5.4 Describe processes in finding solutions; Suggest alternate strategies				
5.5 Solve simple logic problems				
5.6 Use models or "act out" to solve problems				
5.7 Use drawings/ diagrams/ organized lists to solve problems				
5.8 Use guess and check to solve problems				

Goal 6: The learner will demonstrate an understanding of data collection, display, and interpretation.

6.1 Collect, sort, organize, display information				
6.2 Summarize/ interpret information on charts/ graphs; Make predictions				
6.3 Collect and display data over a period of time				
6.4 Locate points on a number line; Explore positions on grids				
6.5 Complete simple experiments; Describe results; Make predictions				

Goal 7: The learner will be able to compute using whole numbers.

7.1 Use manipulatives/ counting strategies to derive addition/ subtraction facts; Write equations				
7.2 Use addition/ subtraction to solve problems				
7.3 Tell missing addends for addition facts				
7.4 Model/ record/ solve 3 single digit addition				
7.5 Model/ record/ explain 2-digit addition/ subtraction				
7.6 Demonstrate ways to verify reasonableness of answers				
7.7 Add/ subtract 2-digit problems with/ without renaming,using manipulatives as needed for accuracy				
7.8 Add/ subtract 1 or 10 mentally				
7.9 Model repeated addition				
7.10 Model sharing equally				
7.11 Memorize addition/ subtraction facts				

Code: *M = Most of the time* *S = Sometimes* *N = Not yet*

NCTM STANDARDS:

Assumptions about Mathematics Programs

The National Council of Teachers of Mathematics has articulated six assumptions upon which the K-4 standards are based. These assumptions are paraphrased and summarized below:

1. The K-4 curriculum should be conceptually oriented.

A primary focus of the curriculum should be on the development of mathematical understandings and relationships. Time devoted to developing a strong conceptual framework promotes problem solving and provides anchoring for skill acquisition.

2. The K-4 curriculum should actively involve children in doing mathematics.

Young children are active individuals who construct and modify ideas and integrate existing knowledge through interacting with the physical world, materials, and other children. Throughout the standards, verbs such as explore, validate, represent, solve, construct, discuss, use, investigate, describe, develop and predict convey this active physical and mental involvement by children.

3. The K-4 curriculum should emphasize the development of children's mathematical thinking and reasoning abilities.

Future uses of and needs for mathematics make the ability to think, reason, and solve problems a primary goal for the study of mathematics. The curriculum must address the goal of developing students who possess confidence in their ability to think and communicate mathematically, to solve problems, to demonstrate flexibility, to make appropriate decisions in selecting strategies and techniques, to recognize familiar mathematical structures in unfamiliar settings, to detect patterns, and to analyze data.

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4. The K-4 curriculum should emphasize the application of mathematics.

In order for children to view mathematics as a practical, useful subject, they must see how it can be applied to a wide variety of real world problems and phenomena. Even as most mathematical ideas of the K-4 curriculum develop from the everyday world, they must be regularly applied to real world situations.

5. The K-4 curriculum should include a broad range of content.

In order to develop mathematical literacy, students need to possess knowledge of such important branches of mathematics as measurement, geometry, statistics, probability, and algebra. The informal approach at this level establishes the foundation for further study and permits children to acquire the additional knowledge they need. These topics are highly appropriate for young learners, for they make important contributions to children's mathematical development and help them to see the usefulness of mathematics. Children need to understand how mathematical ideas are interrelated. Teachers must also provide productive, intriguing activities and applications.

6. The K-4 curriculum should make appropriate and ongoing use of calculators and computers.

Calculators are rapidly being accepted as valuable tools for learning mathematics at all levels of the curriculum. While they do not replace the need to learn basic facts, to compute mentally, or to understand pencil and paper computations, they highlight the importance of recognizing when computed results are reasonable.

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NCTM STANDARDS

STANDARD 1. MATHEMATICS AS PROBLEM SOLVING

In grades K-4, the study of mathematics should emphasize problem solving so that students can:

- use problem solving approaches to investigate and understand mathematical content;
- formulate problems from everyday and mathematical situations;
- develop and apply strategies to solve a wide variety of problems;
- verify and interpret results with respect to the original problem;
- acquire confidence in using mathematics meaningfully.

STANDARD 2. MATHEMATICS AS COMMUNICATION

In grades K-4, the study of mathematics should include numerous opportunities for communication so that students can:

- relate physical materials, pictures and diagrams to mathematical ideas;
- reflect upon and clarify their thinking about mathematical ideas and situations;
- relate their everyday language to mathematical language and symbols;
- realize that representing, discussing, listening, writing, and reading mathematics are a vital part of learning and using mathematics.

STANDARD 3. MATHEMATICS AS REASONING

In grades K-4, the study of mathematics should emphasize reasoning so that students can:

- draw logical conclusions about mathematics;
- use models, known facts, properties, and relationships to explain their thinking;
- justify their answers and solution processes;
- use patterns and relationships to analyze mathematical situations;
- believe that mathematics makes sense.

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STANDARD 4. MATHEMATICAL CONNECTIONS

In grades K-4, the study of mathematics should include opportunities to make connections so that students can:

- link conceptual and procedural knowledge;
- relate various representations of concepts or procedures to one another;
- recognize relationships among different topics in mathematics;
- use mathematics in other curriculum areas;
- use mathematics in their daily lives.

STANDARD 5. ESTIMATION

In grades K-4, the curriculum should include estimation so students can:

- explore estimation strategies;
- recognize when an estimate is appropriate;
- use estimation to determine reasonableness of results;
- apply estimation in working with quantities, measurement, computation, and problem solving.

STANDARD 6. NUMBER SENSE AND NUMERATION

In grades K-4, the mathematics curriculum should include whole number concepts and skills so that students can:

- construct number meanings through real-world experiences and the use of physical materials;
- understand our numeration system by relating counting, grouping,and place value concepts;
- develop number sense;
- interpret the multiple uses of numbers encountered in the real world.

STANDARD 7. CONCEPTS OF WHOLE NUMBER OPERATIONS

In grades K-4, the mathematics curriculum should include concepts of addition, subtraction, multiplication, and division of whole numbers so that students can:

- develop meaning for the operations by modeling and discussing a rich variety of problem situations;

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- relate the mathematical language and symbolism of operations to problem situations and informal language;
- recognize that a wide variety of problem structures can be represented by a single operation;
- develop operation sense.

STANDARD 8. WHOLE NUMBER COMPUTATION

In grades K-4, the mathematics curriculum should develop whole number computation so that students can:

- model, explain, and develop reasonable proficiency with basic facts and algorithms;
- use a variety of mental computation and estimation techniques;
- use calculators in appropriate computational situations;
- select and use computation techniques appropriate to specific problem situations and determine whether the result is reasonable.

STANDARD 9. GEOMETRY AND SPATIAL SENSE

In grades K-4, the mathematics curriculum should include two- and three- dimensional geometry so that students can:

- describe, model, draw, and classify shapes;
- investigate and predict results of combining, subdividing, and changing shapes;
- develop spatial sense;
- relate geometric ideas to number and measurement ideas;
- recognize and appreciate geometry in their world.

STANDARD 10. MEASUREMENT

In grades K-4, the mathematics curriculum should include measurement so that students can:

- understand the attributes of length, capacity, weight, area, volume, time, temperature, and angle;
- develop the process of measuring and concepts related to units of measurement;
- make and use estimates of measurement;
- make and use measurements in problem and everyday situations.

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STANDARD 11. STATISTICS AND PROBABILITY

In grades K-4, the mathematics curriculum should include experiences with data analysis and probability so that students can:

- collect, organize, and describe data;
- construct, read, and interpret displays of data;
- formulate and solve problems that involve collecting and analyzing data;
- explore concepts of chance.

STANDARD 12. FRACTIONS AND DECIMALS

In grades K-4, the mathematics curriculum should include fractions and decimals so that students can:

- develop concepts of fractions, mixed numbers and decimals;
- develop number sense for fractions and decimals;
- use models to relate fractions to decimals and to find equivalent fractions;
- use models to explore operations on fractions and decimals;
- apply fractions and decimals in problem situations.

STANDARD 13. PATTERNS AND RELATIONSHIPS

In grades K-4, the mathematics curriculum should include patterns and relationships so that students can:

- recognize, extend, describe, and create a wide variety of patterns;
- represent and describe mathematical relationships;
- explore the use of variables and open sentences to express relationships.

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Curriculum and Evaluation Standards for School Mathematics
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ESSENTIAL MATHEMATICS FOR THE 21ST CENTURY

The National Council of Supervisors of Mathematics has described twelve components of mathematics which are essential for all students in preparation for living and working in the 21st century:

1. Problem Solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-text problems. Problem solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. Students should see alternate solutions of problems; they should experience problems with more than a single solution.

2. Communicating Mathematical Ideas

Students should learn the language and notation of mathematics. For example, they should understand place value and scientific notation. They should learn to receive mathematical ideas through listening, reading, and visualizing. They should be able to present mathematical ideas by speaking, writing, drawing pictures and graphs, and demonstrating with concrete models. They should be able to discuss mathematics and ask questions about mathematics.

3. Mathematical Reasoning

Students should learn to make independent investigations of mathematical ideas. They should be able to identify and extend patterns and use experiences and observations to make conjectures (tentative conclusions). They should learn to use a counter-example to disprove a conjecture, and they should learn to use models, known facts, and logical arguments to validate a conjecture. They should be able to distinguish between valid and invalid arguments.

4. Applying Mathematics to Everyday Situations

Students should be encouraged to take everyday situations, translate them into mathematical representations (graphs, tables, diagrams or mathematical expressions), process the mathematics, and interpret the results in light of the initial situation. They should be able to solve ratio, proportion, percent, direct variation and inverse variation problems. Not only should students see how mathematics is applied in the real world, but they should observe how mathematics grows from the world around them.

5. Alertness to the Reasonableness of Results

In solving problems, students should question the reasonableness of a solution or conjecture in relation to the original problem. Students must develop the number sense to determine if results of calculations are reasonable in relation to the original numbers and the operations used. With the increase in the use of calculating devices in society, this capability is more important than ever.

6. Estimation

Students should be able to carry out rapid approximate calculations through the use of mental arithmetic and a variety of computational estimation techniques. When computation is needed in a problem or consumer setting, an estimate can be used to check reasonableness, examine a conjecture, or make a decision. Students should acquire simple techniques for estimating measurements such as length, area, volume and mass (weight). They should be able to decide when a particular result is precise enough for the purpose at hand.

7. Appropriate Computational Skills

Students should gain facility in using addition, subtraction, multiplication, and division with whole numbers and decimals. Today, long, complicated computations should be done with a calculator or computer. Knowledge of single-digit number facts is essential, and using mental arithmetic is a valuable skill. In learning to apply computation, students should have practice in choosing the appropriate computational method: mental arithmetic, paper-pencil algorithm, or calculating device. Moreover, there are everyday situations that demand recognition of, and simple computation with, common fractions. In addition, the ability to recognize, use, and estimate with percents must also be developed and maintained.

8. Algebraic Thinking

Students should learn to use variables (letters) to represent mathematical quantities and expressions; they should be able to represent mathematical functions and relationships using tables, graphs, and equations. They should understand and correctly use positive and negative numbers, order of operations, formulas, equations and inequalities. They should recognize the ways in which one quantity changes in relation to another.

9. Measurement

Students should learn the fundamental concepts of measurement through concrete experiences. They should be able to measure distance, mass (weight), time, capacity, temperature, and angles. They should learn to calculate simple perimeters, areas and volumes. They should be able to perform measurement in both metric and customary systems using the appropriate tools and levels of precision.

10.Geometry

Students should understand the geometric concepts necessary to function effectively in the three-dimensional world. They should have knowledge of concepts such as parallelism, perpendicularity, congruence, similarity and symmetry. Students should know properties of simple plane and solid geometric figures. Students should visualize and verbalize how objects move in the world around them using terms such as slides, flips and turns. Geometric concepts should be explored in settings that involve problem solving and measurement.

11.Statistics

Students should plan and carry out the collection and organization of data to answer questions in their everyday lives. Students should know how to construct, read, and draw conclusions from simple tables, maps, charts, and graphs. They should be able to present information about numerical data such as measures of central tendency (mean, median, mode) and measures of dispersion (range, deviation). Students should recognize the basic uses and misuses of statistical representation and inference.

12.Probability

Students should understand elementary notions of probability to determine the likelihood of future events. They should identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election results, business forecasts, and outcomes of sporting events. They should learn how probability applies to research results and to the decision-making process.

Position paper of The National Council of Supervisors of Mathematics
Essential Mathematics Task Force, June 1988

MATHEMATICS DIVISION STAFF DIRECTORY

Raleigh Office:	Phone Number:
Robert R. Jones, Director	(919) 733-3602
Cleo Meek, Assistant Director	(919) 733-3602
Jeane Joyner, Elementary Consultant	(919) 733-3602
Mildred Quick, Secretary	(919) 733-3602
Marie Robinson, Secretary	(919) 733-3602

Regional Mathematics Coordinators:

Region 1, Williamston - Barbara Hardison	(919) 792-5166
Region 2, Jacksonville - Carol Mazingo	(919) 455-8100
Region 3, Zebulon - Ann Hart	(919) 269-7438
Region 4, Carthage - Yvonne Jordan	(919) 947-5871
Region 5, Greensboro - Barbara Leland	(919) 334-5764
Region 6, Charlotte - Sandi Lee	(704) 392-0378
Region 7, North Wilkesboro - Carl Richardson	(919) 667-2191
Region 8, Waynesville - Mary Joyce Henderson	(704) 452-0363